(سارا اسدی راد)

① $A = 180 - (B+C) = 180 - 120 = 60$ → $\hat{A}$

$\frac{BC}{\sin A} = \frac{AC}{\sin B} \rightarrow \frac{20}{\sin 60} = \frac{20\sqrt{3}}{\sin B} \rightarrow \sin B = \frac{1}{2} \rightarrow \hat{B} = 30$

$\hat{C} = 120 - 30 = 90$

② $\frac{2\sqrt{2}}{\sin 45} = \frac{AC}{\sin 60} \rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{3}}{2}} \rightarrow AC = 2\sqrt{3}$

$\frac{2\sqrt{3}}{\sin 90} = \frac{x}{\sin 60} \rightarrow \frac{2\sqrt{3}}{1} = \frac{x}{\frac{\sqrt{3}}{2}} \rightarrow x = \frac{\sqrt{3}}{2} \times 2\sqrt{6} = 3$

③ $\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow c = b\frac{\sin C}{\sin B}$ $\sim (b^2-2)\frac{\sin C}{\sin B} = b\frac{\sin C}{\sin B}$

$\rightarrow b^2 - 2 = b \rightarrow b^2 - b - 2 = 0 \rightarrow (b-2)(b+1) = 0$

$b = 2$ ✓ , $b = -1$ ✗ (غ ق ق)

④ $b^2 = a^2 + c^2 - 2ac\cos B$ جایگذاری ✓ $\rightarrow (13)^2 = (21)^2 + (20)^2 - 2(21)(20)\cos B$

$a = BC = 21$ $c = AB = 20$ $b = AC = 13$

$\hookrightarrow \cos B = 0.8$

$\sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - 0.64} = 0.6$

⑤ $a^2 = b^2 + c^2 - 2bc\cos A \rightarrow a^2 = (4)^2 + (2(1+\sqrt{3}))^2 - 2(4)(2(1+\sqrt{3}))\cos 60$

$\rightarrow a = \sqrt{24} = 2\sqrt{6}$

الف)

ب)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{2\sqrt{6}}{\sin 60} = \frac{4}{\sin B} \rightarrow \sin B = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45$

$\hat{C} = 180 - (60 + 45) = 75$

⑥

(رسا امینی راد)

۷) الف)

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$

$\to BC^2 = (8)^2 + (5)^2 - 2(8)(5)\cos 60 \to \boxed{BC = 7}$

ب)

$S = \frac{1}{2} ab \sin C \to \frac{1}{2}(AB)(AC)\sin A \to \frac{1}{2}(8)(5)\sin 60 = \boxed{10\sqrt{3}}$

۸)

$a^2 = b^2 + c^2 - 2bc\cos A \xrightarrow{\text{جایگذاری}} (b+c)^2 - a^2 = (b^2 + 2bc + c^2) - (b^2 + c^2 - 2bc\cos A)$

$\to \frac{2bc(1+\cos A)}{bc(\cos A + 1)} = \boxed{2}$

۹)

$a^3 + b^3 - c^3 = a^2(a+b-c) \to a^3 + b^3 - c^3 = a^3 + a^2 b - a^2 c$

$\to (b-c)(b^2 + bc + c^2) = a^2(b-c) \to b^2 + bc + c^2 = a^2$

قانون کسینوس $\to a^2 = b^2 + c^2 - 2bc\cos A \to b^2 + c^2 - 2bc\cos A = b^2 + bc + c^2$

$\cos A = -\frac{1}{2} \to$ بنابراین $\boxed{A = 120}$

۱۰)

$a^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$

$a^2 = (3 - \cos\theta)^2 + (1 + 3\cos\theta)^2 - 2(3 - \cos\theta)(1 + \cos\theta)\frac{\sqrt{3}}{2}$

$\to a^2 = 10 - 3\sqrt{3} - 8\sqrt{3}\cos\theta + (10 + 3\sqrt{3})\cos^2\theta$

$S = \frac{1}{2}(AB)(AC)\sin A = 2 \to \frac{1}{2}(3 - \cos\theta)(1 + 3\cos\theta) = 4 \to \cos\theta = 1$

$\boxed{a^2 = 10 - 8\sqrt{3}}$