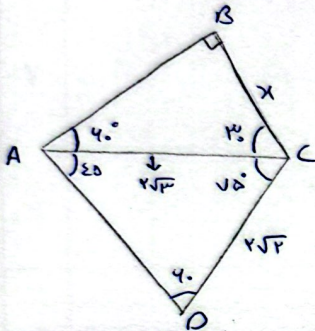


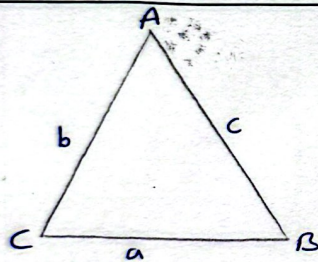
$$\hat{B} + \hat{C} = 120^\circ \rightarrow \hat{A} = 180^\circ - 120^\circ = 60^\circ$$

$$\frac{\sin 4^\circ}{10} = \frac{\sin \hat{B}}{\frac{10\sqrt{3}}{14}} \rightarrow \sin \hat{B} = \frac{\sqrt{3}}{14} \rightarrow \hat{B} = 4^\circ \rightarrow \hat{C} = 120^\circ - 4^\circ = 116^\circ$$



$$\frac{DC}{\sin 60^\circ} = \frac{AC}{\sin 4^\circ} \rightarrow \frac{10\sqrt{3}/14}{\frac{\sqrt{3}}{2}} = \frac{AC}{\frac{\sqrt{3}}{14}} \rightarrow AC = 10\sqrt{3}$$

$$BC = x = \frac{\sqrt{3}}{14} \times 10\sqrt{3} = 10$$

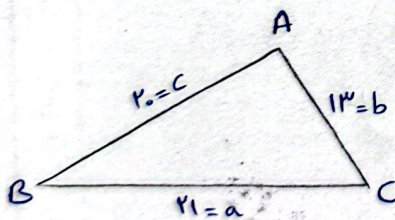


$$(b^2 - r^2) \frac{\sin \hat{C}}{\sin \hat{B}} = c \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\rightarrow b \frac{\sin C}{\sin B} = c \rightarrow b^2 - r^2 = b \rightarrow b^2 - b - r^2 = 0$$

$$(b+1)(b-r) = 0$$

$$\rightarrow b = -1 \text{ (not possible)} \rightarrow b = r$$



$$a^2 = b^2 + c^2 - 2bc \cos A$$

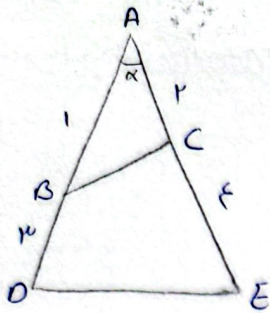
$$\rightarrow 14^2 = 10^2 + 10^2 - 2 \cdot 10 \cdot 10 \cdot \cos 60^\circ \rightarrow \cos 60^\circ = \frac{1}{2}$$

$$\rightarrow \sin^2 B + \cos^2 B = 1 \rightarrow \sin^2 B = \frac{1}{4} \rightarrow \sin B = \frac{1}{2} \rightarrow \hat{B} = 30^\circ$$

$$\text{الف) } a^2 = 14^2 + 10^2 + 10^2 - 2 \cdot 14 \cdot 10 \cdot \cos 120^\circ \rightarrow a^2 = 14^2 + 10^2 + 10^2 - 2 \cdot 14 \cdot 10 \cdot (-\frac{1}{2}) = 256 \rightarrow a = 16$$

$$\text{ب) } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{16}{\sin 120^\circ} = \frac{10}{\sin B} \rightarrow \sin B = \frac{10 \sin 120^\circ}{16} \rightarrow \hat{B} = 48^\circ$$

$$\rightarrow \hat{C} = 180^\circ - 120^\circ - 48^\circ = 12^\circ$$



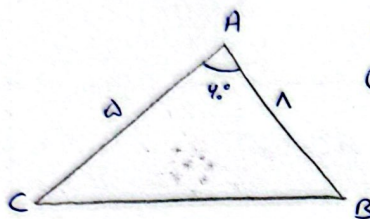
$$a^2 = b^2 + c^2 - 2bc \cos \hat{A}$$

$$E = 2 + 1 - E \cos \alpha \rightarrow \cos \alpha = \frac{1}{E} \checkmark$$

$$(DE)^2 = 14 + 14 - 2 \times 1 \times \left(\frac{1}{E}\right) = 1$$

$$DE = \sqrt{1} \checkmark$$

(y)



$$a^2 = b^2 + c^2 - 2bc \cos \hat{A}$$

$$(BC)^2 = 4 + 4 - 2 \times \left(\frac{1}{\sqrt{2}}\right) = 2$$

$$BC = \sqrt{2} \checkmark$$

$$S = \frac{1}{2} bc \sin \hat{A} = \frac{1}{2} \times \sqrt{2} \times \sqrt{2} = 1 \checkmark$$

(y)

$$\frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} = \frac{b^2 + c^2 + 2bc - a^2 - b^2 - c^2 + 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} = \frac{2bc(\cos \hat{A} + 1)}{bc(\cos \hat{A} + 1)} = 2 \checkmark$$

(y)

$$\frac{a^2 + b^2 - c^2}{a + b - c} = a^2 \rightarrow a^2 + b^2 - c^2 = a^2 + a^2 b - a^2 c \rightarrow b^2 - c^2 = a^2 b - a^2 c$$

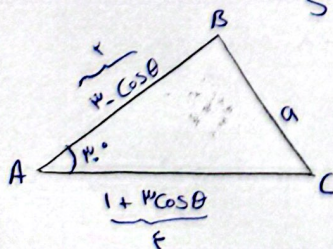
$$(b-c)(b^2 + c^2 - bc) = a^2(b-c)$$

$$a^2 = b^2 + c^2 - bc = b^2 + c^2 - 2bc \cos \hat{A}$$

$$2bc \cos \hat{A} = 1 \rightarrow \cos \hat{A} = \frac{1}{2}$$

$$2 \cos \hat{A} = -1$$

$$\cos \hat{A} = -\frac{1}{2} \rightarrow \hat{A} = 120^\circ$$



$$S = \frac{1}{2} (1 + 2 \cos \theta) (1 - \cos \theta) \times \frac{1}{2} = 1$$

$$-2 \cos \theta + 2 \cos \theta - 1 = 0 \rightarrow \cos \theta = 1$$

$$\cos \theta = \frac{1}{2} \times 2 \cos \theta$$

$$a^2 = b^2 + c^2 - 2bc \cos \theta$$

$$a^2 = 14 + 14 - (2 \times 1 \times 1 \times \frac{\sqrt{2}}{2}) = 12 - \sqrt{2} \checkmark$$

(y)