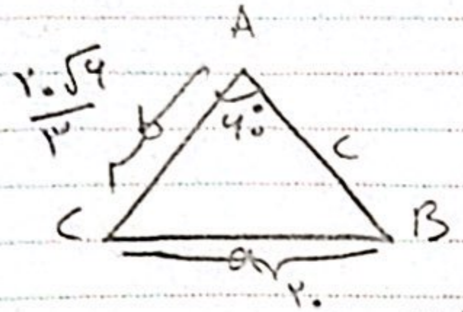
هلنا محمودی، دوازدهم جمعه

①:



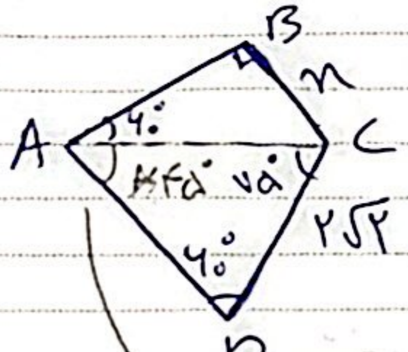
$B + C = 120^\circ \Rightarrow A = 60^\circ$

$\frac{b}{\sin B} = \frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \frac{20}{\sin 60^\circ = \frac{\sqrt{3}}{2}} = \frac{\frac{20\sqrt{6}}{3}}{\sin B} \Rightarrow \sin B = \frac{\frac{\sqrt{18}}{6}}{1} = \frac{\cancel{\sqrt{18}}^{\,3\sqrt{2}}}{\cancel{6}_{\,2}} \Rightarrow \sin B = \frac{\sqrt{2}}{2} \Rightarrow B = 45^\circ$

$\underset{45^\circ}{B} + C = 120^\circ \Rightarrow \underline{\hat{C} = 75^\circ}\ ,\ \underline{\hat{B} = 45^\circ}$

②:

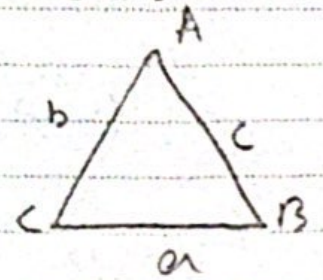


$\Rightarrow \frac{\cancel{2\sqrt{2}}^{\,4}}{\sin 45^\circ = \frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{3}}{2} = \sin 60^\circ} \Rightarrow \underline{AC = 2\sqrt{3}}$

$180 - (\overset{135}{75 + 60}) = 45^\circ$

$\Rightarrow \frac{2\sqrt{3}}{\sin 90^\circ = 1} = \frac{x}{\frac{\sqrt{3}}{2}} \Rightarrow \boxed{x = 3}$ //

۲۰ رمضان ۱۴۳۹



۳:

$$\begin{cases}(b^2-2)\dfrac{\sin\hat{C}}{\sin\hat{B}}=c\\[2ex] \dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}\end{cases}$$

$$\Rightarrow b\frac{\sin C}{\sin B}=c \Rightarrow b^2-2=b \Rightarrow \underset{a}{1}b^2\,\underset{b}{-1}b\,\underset{c}{-2}=0$$

$$\xrightarrow{a+c=b} \quad \underset{\text{غ ق ق}}{b=-1} \quad , \quad \underline{b=\frac{-c}{a}=2 \ \text{ق ق}}$$

۴:

$$a^2=b^2+c^2-2bc\cos A \xrightarrow{\text{با توجه به صورت سوال}}$$

$$169=441+400-840(\cos\hat{B}) \longrightarrow \cos\hat{B}=\frac{672}{840}=\frac{4}{5}$$

$$\Rightarrow \cos^2 B+\sin^2 B=1 \Rightarrow \frac{16}{25}+\sin^2 B=1 \longrightarrow \sin^2 B=\frac{9}{25}$$

$$\Rightarrow \sin\hat{B}=\frac{3}{5}$$

۵:

الف)
$$a^2=16+4(1+3+2\sqrt{3})-(\cancel{16}(1+\sqrt{3})\frac{1}{\cancel{2}})$$

$$\Rightarrow a^2=16+4+12+\cancel{8\sqrt{3}}-8-\cancel{8\sqrt{3}}=24 \Rightarrow \boxed{a=2\sqrt{6}}$$ ✓ جواب

$a^2=24$

ب) $\frac{b}{\sin\hat B} = \frac{c}{\sin\hat C} = \frac{a}{\sin\hat A} \Rightarrow \frac{2\sqrt6}{\frac{\sqrt3}{2}} = \frac{4}{\sin\hat B}$

$\Rightarrow \sin\hat B = \frac{\sqrt2}{2} \longrightarrow \hat B = 45^\circ$ پس $\Rightarrow A+B+C=180^\circ \Rightarrow \underline{C=75^\circ}$

⑥ ، طبق قضیه‌ی کسینوس‌ها $\Rightarrow a^2 = b^2 + c^2 - 2bc\cos\hat A$

با توجه به شکل سوال : $\rightarrow 4 = \overset{5}{4+1} - 4\cos\alpha \Rightarrow \cos\alpha = \frac{1}{4}$

$DE^2 = 36 + 16 - 48(\underbrace{\cancel{\cos\alpha}}_{\frac14}) \Rightarrow \boxed{DE = 2\sqrt{10}}$ ✓

⑦ : الف) $a^2 = b^2 + c^2 - 2bc\cos\hat A$

$\rightarrow (BC)^2 = 25 + 64 - 80 \times \frac{1}{2} \Rightarrow (BC)^2 = 49 \Rightarrow \boxed{BC = 7}$ ✓

ب) $S_{\text{مثلث}} = \frac{1}{2} bc \times \sin\hat A \longrightarrow \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt3}{2} = \boxed{10\sqrt3}$ ✓

۸)

$$\frac{(b+c)^2-a^2}{bc(\cos\hat{A}+1)}=\frac{b^2+2bc+c^2-\cancel{c^2}-\cancel{b^2}+2bc\cos\hat{A}}{bc(\cos\hat{A}+1)}$$

$$\Rightarrow \frac{2bc+2bc\cos\hat{A}}{bc(\cos\hat{A}+1)}=\frac{2\cancel{bc(\cos\hat{A}+1)}}{\cancel{bc(\cos\hat{A}+1)}}=2 \checkmark$$

۹)

$$\frac{a^3+b^3-c^3}{a+b-c}=a^2 \Rightarrow \cancel{a^3}+b^3-c^3=\cancel{a^3}+a^2b-a^2c$$

$$\Rightarrow b^3-c^3=a^2b-a^2c \Rightarrow \cancel{(b-c)}(b^2+c^2+bc)=a^2\cancel{(b-c)}$$

$$\Rightarrow \cancel{b^2}+\cancel{c^2}+bc=a^2=\cancel{b^2}+\cancel{c^2}-2bc\cos\hat{A}$$

$$\Rightarrow -2\cos\hat{A}=1 \Rightarrow \boxed{\cos\hat{A}=\frac{-1}{2}} \checkmark$$

۱۰)

$$\frac{1}{2}\times(1+2\cos\theta)(3-\cos\theta)\times\frac{1}{2}=2$$

$$\Rightarrow -2\cos\theta+8\cos\theta-5=0 \Longrightarrow (1)\ \cos\theta=1$$

$$(2)\ \cos\theta=\frac{5}{2}$$ غ ق ق

زیرا $-1\le\cos\theta\le 1$

$$a^2=b^2+c^2-2bc\cos\hat{A}$$

$$a^2=16+4-8\sqrt{3} \longrightarrow \boxed{a^2=20-8\sqrt{3}} \checkmark$$