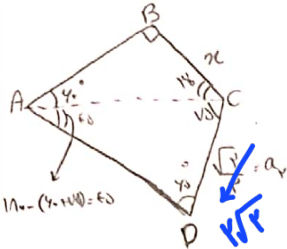

 $\hat{B} + \hat{C} = 110^\circ \rightarrow \hat{A} = 70^\circ$
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{10}{\sin 40} = \frac{10\sqrt{2}}{\sin 70} \rightarrow \sin 70 = \frac{10\sqrt{2}}{10} \times \frac{\sin 40}{1} = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$
 $= \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \rightarrow \sin B = \frac{\sqrt{2}}{2} \rightarrow \boxed{B = 45^\circ}$ $\hat{B} + \hat{C} = 110^\circ \rightarrow 45 + \hat{C} = 110^\circ \rightarrow \boxed{\hat{C} = 65^\circ}$

(۲)

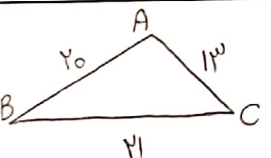

 $\frac{d}{\sin D} = \frac{a_r}{\sin A_r} \rightarrow \frac{d}{\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \rightarrow d = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$
 $\rightarrow \mathcal{K} = 90^\circ$ (مربع) $\mathcal{K} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \rightarrow \mathcal{K} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

(۱)

$\frac{\sin \hat{B}^r}{b} = \frac{\sin \hat{C}^r}{c} = k \rightarrow \frac{\sin \hat{C}^r}{\sin \hat{B}^r} = \frac{c}{b}$

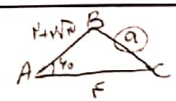
$(b^r - r) \frac{\sin \hat{C}^r}{\sin \hat{B}^r} = (b^r - r) \frac{c}{b} \xrightarrow{b \neq 0} (b^r - r) \frac{c}{b} = c$

$\frac{b^r - r}{b} = 1 \rightarrow b^r - b - r = 0 \rightarrow (b - r)(b + 1) = 0 \xrightarrow{b \neq -1} \boxed{b = r}$


 $AC^2 = AB^2 + BC^2 - 2(AB)(BC) \cos \hat{B}$
 $149 = 100 + 100 - 2(10)(10) \cos \hat{B} \rightarrow 149 = 100 - 100 \cos \hat{B}$
 $\rightarrow 100 \cos \hat{B} = 49 \rightarrow \cos \hat{B} = \frac{49}{100} = \frac{7}{10}$
 $\rightarrow \cos^2 \hat{B} + \sin^2 \hat{B} = 1 \rightarrow \frac{49}{100} + \sin^2 \hat{B} = 1 \rightarrow \sin^2 \hat{B} = \frac{51}{100} \rightarrow \sin \hat{B} = \frac{\sqrt{51}}{10}$
 $\boxed{\sin \hat{B} = 0.74}$

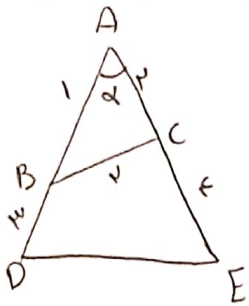
(۲)

الف) $a^r = b^r + c^r - 2bc \cos 40 = 14 + 10 + 10 - 2(10)(10) \cos 40 = 24 - 20 \cos 40 = 24 - 20 \times \frac{4}{5} = 24 - 16 = 8 \rightarrow \boxed{a = \sqrt{8}}$



ب) $\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{\sqrt{8}}{\sin 40} = \frac{10}{\sin 70} \Rightarrow \frac{\sqrt{8}}{\frac{4}{5}} = \frac{10}{\frac{\sqrt{2}}{2}} \Rightarrow \sqrt{8} = \frac{10 \times 4}{5 \times \sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$
 $\rightarrow \hat{C} = 180 - (45 + 40) = 95^\circ \rightarrow \boxed{\hat{C} = 95^\circ}$

(۲)



$$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos \alpha \rightarrow 1 = 1 + 1 - 2 \cos \alpha \rightarrow \cos \alpha = \frac{1}{2} \checkmark$$

$$\rightarrow DE^2 = AD^2 + AE^2 - 2(AD)(AE) \cos \alpha \rightarrow DE^2 = \frac{1}{4} + \frac{1}{4} - 2(\frac{1}{4})(\frac{1}{4}) = \frac{1}{4}$$

$$\rightarrow DE = \frac{1}{2} \rightarrow DE = \sqrt{\frac{1}{4}} = \frac{1}{2} \checkmark$$

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$$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos 60^\circ \rightarrow BC^2 = 4^2 + 4^2 - 2(4)(4)(\frac{1}{2}) = 16 - 16 = 0 \rightarrow BC = 0 \checkmark$$

$$S_{\triangle ABC} = \frac{1}{2} \times AB \times AC \times \sin 60^\circ = \frac{1}{2} \times 4 \times 4 \times \frac{\sqrt{3}}{2} = 4\sqrt{3} \checkmark$$

(1)

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$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc \cos A)}{bc(\cos A + 1)} = \frac{2bc + 2bc \cos A}{bc(\cos A + 1)}$$

$$\frac{2bc(1 + \cos A)}{bc(\cos A + 1)} = 2 \checkmark$$

8

$$a^2 + b^2 - c^2 = a^2(a+b-c) = a^2 + a^2(b-c) \rightarrow b^2 - c^2 = a^2(b-c)$$

$$(b-c)(b^2 + c^2 + bc) = a^2(b-c)$$

$$a^2 = b^2 + c^2 + bc \rightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = -\frac{1}{2} \rightarrow A = 120^\circ$$

9

$$S_{\triangle ABC} = \frac{1}{2} \times AB \times AC \times \sin \theta = \frac{1}{2} \times (1 - \cos \theta)(1 + \cos \theta) \times \frac{1}{2} = \frac{1 - \cos^2 \theta}{4} = \frac{\sin^2 \theta}{4} = \frac{1}{4}$$

$$\rightarrow -\cos^2 \theta + \cos \theta + \frac{1}{2} = 0 \rightarrow -\cos^2 \theta + \cos \theta - \frac{1}{2} = 0 \rightarrow \cos \theta = \frac{-1 \pm \sqrt{1 - 2}}{-2} = \frac{-1 \pm i}{-2}$$

$$\rightarrow AB = 1 - (-1) = 2, AC = 1 + (-1) = 0$$

$$a^2 = b^2 + c^2 - 2bc \cos \theta \rightarrow a^2 = 1 + 1 - 2(1)(1)(\frac{\sqrt{3}}{2}) = 2 - \sqrt{3}$$

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