نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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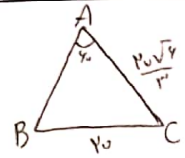

$\hat{B}+\hat{C}=120^\circ \rightarrow \hat{A}=60^\circ$

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \Rightarrow \frac{20}{\frac{\sqrt{3}}{2}}=\frac{\frac{20\sqrt{2}}{3}}{\sin B} \rightarrow \sin B=\frac{\frac{10\sqrt{2}}{3}\times\frac{\sqrt{3}}{2}}{20}=\frac{\sqrt{6}}{4}=\frac{\sqrt{2}\times\sqrt{3}}{\sqrt{2}\times\sqrt{2}}$

$=\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2} \rightarrow \sin B=\frac{\sqrt{2}}{2} \rightarrow \boxed{B=45^\circ}$ $\quad \hat{B}+\hat{C}=120^\circ \rightarrow 45+\hat{C}=120^\circ \rightarrow \boxed{\hat{C}=75^\circ}$

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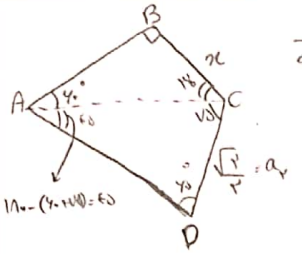

$\frac{d}{\sin D}=\frac{a_2}{\sin A_2} \rightarrow \frac{d}{\frac{\sqrt{3}}{2}}=\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \rightarrow \boxed{d=\frac{\sqrt{3}}{2}}$ = وتر

$\rightarrow x=$ ضلع روبه‌رو زاویه 60° ، $x=\frac{\sqrt{3}}{2}\times$ وتر $\rightarrow x=\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}=\boxed{\frac{\sqrt{6}}{4}}$

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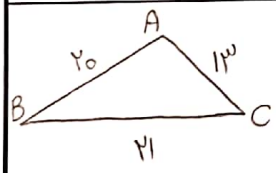

$AC^2=AB^2+BC^2-2(AB)(BC)\cos\hat{B}$

$149=400+441-2(20)(21)\cos\hat{B} \rightarrow 149=841-840\cos B$

$\rightarrow 840\cos\hat{B}=692 \rightarrow \cos\hat{B}=\frac{692}{840}=\frac{8}{10}$

$\rightarrow \cos^2 B+\sin^2 B=1 \rightarrow \frac{64}{100}+\sin^2 B=1 \rightarrow \sin^2 B=\frac{36}{100} \rightarrow \boxed{\sin\hat{B}=0/6}$

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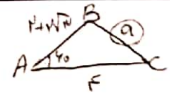
الف) $a^2=b^2+c^2-2bc\cos 60 = 16+4+12+8\sqrt{3}-(2)(4)(2+\sqrt{3})(\frac{1}{2}) = 24 \rightarrow \boxed{a=\sqrt{24}}$



ب) $\frac{a}{\sin A}=\frac{b}{\sin B} \Rightarrow \frac{\sqrt{24}}{\frac{\sqrt{3}}{2}}=\frac{4}{x} \Rightarrow \frac{\sqrt{24}}{\frac{\sqrt{3}}{2}}=2\sqrt{8}=\frac{4}{x} \rightarrow x=\frac{4}{2\sqrt{8}}=\frac{2}{2\sqrt{2}}=\frac{\sqrt{2}}{2} \rightarrow \hat{B}=45^\circ$

$\rightarrow \hat{C}=180-(45+60)=75^\circ \rightarrow \boxed{\hat{C}=75^\circ}$

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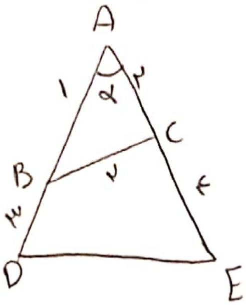

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\alpha \rightarrow 4 = 4 + 1 - 4\cos\alpha \rightarrow \cos\alpha = \frac{1}{4}$

$\rightarrow DE^2 = AE^2 + AD^2 - 2(AD)(AE)\cos\alpha \rightarrow DE^2 = 36 + 16 - \overset{12}{\cancel{48}}\left(\frac{1}{4}\right) = 40$

$\rightarrow DE^2 = 40 \rightarrow DE = \sqrt{40} = \boxed{2\sqrt{10}}$

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الف) $BC^2 = AB^2 + AC^2 - 2(AC)(AB)\cos 60 \rightarrow BC^2 = 64 + 25 - 2(8)(5)\left(\frac{1}{2}\right) = 89 - 40 = 49$

$\rightarrow BC = \sqrt{49} = \boxed{7}$

ب) $S_{\overset{\triangle}{ABC}} = \frac{1}{2} \times AB \times AC \times \sin 60 = \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt{3}}{2} = \boxed{10\sqrt{3}}$

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$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A} + 1)} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc\cos\hat{A})}{bc(\cos\hat{A} + 1)} = \frac{\cancel{b^2} + \cancel{c^2} + 2bc - \cancel{b^2} - \cancel{c^2} + 2bc\cos\hat{A}}{bc(\cos\hat{A} + 1)}$

$\frac{2bc(1 + \cos\hat{A})}{bc(\cos\hat{A} + 1)} = \boxed{2}$

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$S_{\overset{\triangle}{ABC}} = \frac{1}{2} \times AB \times AC \times \sin 30 \rightarrow \frac{1}{2} \times (3 - \cos\theta)(1 + 3\cos\theta) \times \frac{1}{2} = \frac{3 + 9\cos\theta - \cos\theta - 3\cos^2\theta}{4} = 2$

$\rightarrow -3\cos^2\theta + 8\cos\theta + 3 = 8 \rightarrow -3\cos^2\theta + 8\cos\theta - 5 = 0 \rightarrow \cos\theta = \frac{-8 \pm \sqrt{64 - 60}}{-6} = \frac{-8 \pm 2}{-6}$ ↗ $\boxed{+1}$ ق ق ↘ $+\frac{10}{6}$ غ ق ق

$\rightarrow AB = 3 - (+1) = \textcircled{2}$ و $AC = 1 + 3(+1) = \textcircled{4}$

$a^2 = b^2 + c^2 - 2bc\cos 30 \rightarrow a^2 = 4 + 16 - 2(2)(4)\left(\frac{\sqrt{3}}{2}\right) = \boxed{20 - 8\sqrt{3}}$