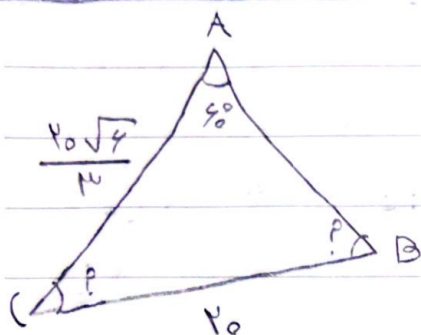


تکلیف شماره ۱۳

به نام خدا ۲۰ نیاش زندی

1

با توجه به قضیه سینوس داریم:



$$\frac{BC}{\sin A} = \frac{AC}{\sin B} \rightarrow \frac{\frac{20}{\sqrt{3}}}{\sin 60^\circ} = \frac{20}{\sin B}$$

$$\frac{20}{\sqrt{3}} = \frac{20}{\sin B}$$

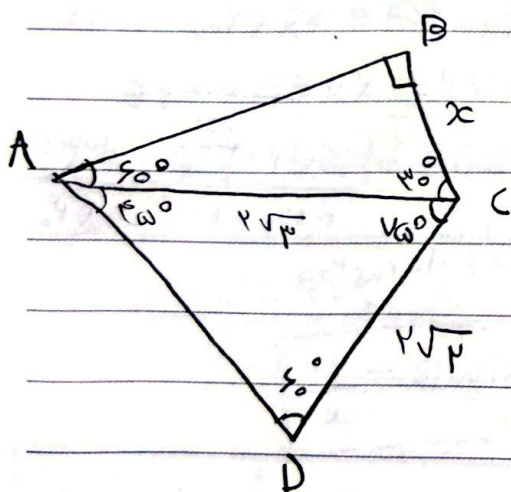
$$\sin B = \frac{20\sqrt{18}}{20} = \frac{\sqrt{18}}{1} = \frac{3\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$$\sin B = \frac{\sqrt{2}}{2} \Rightarrow B = 45^\circ$$

$$180 - 60 - 45 = 75^\circ$$

$$B = 45^\circ \text{ و } C = 75^\circ$$

2



$$\frac{DC}{\sin 60^\circ} = \frac{AC}{\sin 30^\circ}$$

$$\frac{\frac{20}{\sqrt{3}}}{\frac{\sqrt{3}}{2}} = \frac{AC}{\frac{1}{2}} \rightarrow \frac{20}{\sqrt{3}} \cdot \frac{2}{\sqrt{3}} = AC \cdot 2$$

$$AC = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$$

$$\sin 30^\circ = \frac{\sqrt{3}}{2} = \frac{x}{AC} = \frac{x}{10\sqrt{3}}$$

$$\frac{x}{10\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$x = \frac{10\sqrt{3} \times \sqrt{3}}{2} = 15$$

طبق قانون سینوس ها

$$\frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{\sin C}{\sin B} = \frac{c}{b} \quad I$$

$$\frac{\sin C}{\sin B} = \frac{c}{b^2 - 1} \quad II$$

$$I, II \Rightarrow \frac{c}{b} = \frac{c}{b^2 - 1}$$

$$b^2 - 1 = b$$

$$b^2 - b - 1 = 0$$

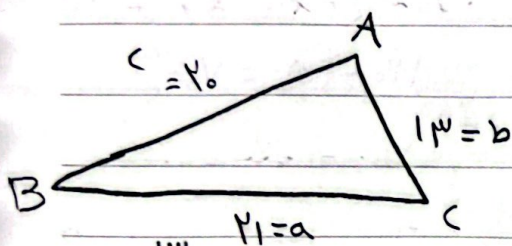
$$(b-1)(b+1) = 0$$

$$b = -1$$

$$b = 1$$

غلق

طبق قانون کسینوس ها



$$\begin{array}{r} 11 \\ \times 13 \\ \hline 166 \\ 121 \\ \hline 143 \end{array}$$

$$\begin{array}{r} 11 \\ \times 13 \\ \hline 166 \\ 121 \\ \hline 143 \end{array}$$

$$b^2 = a^2 + c^2 - 2(ac \cos B)$$

$$13^2 = 11^2 + 10^2 - 2(11 \times 10 \times \cos B)$$

$$169 = 121 + 100 - 2(110 \cos B)$$

$$169 = 221 - 2(110 \cos B)$$

$$221 - 169 = 2 \times 110 \times \cos B$$

$$52 = 220 \cos B$$

$$\cos B = \frac{52}{220} = \frac{13}{55}$$

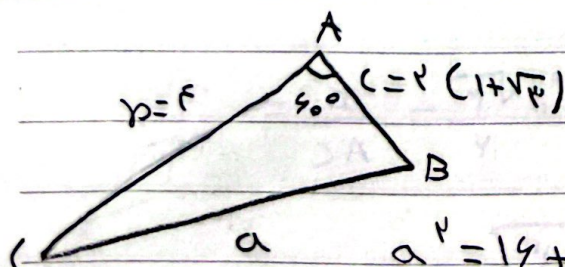
$$\sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - \left(\frac{13}{55}\right)^2} = \frac{52}{55}$$

$$\sin B = 1 - \cos B$$

$$\sin^2 B = 1 - \frac{169}{220}$$

$$\sin^2 B = \frac{9}{220}$$

$$\sin B = \frac{3}{\sqrt{220}}$$



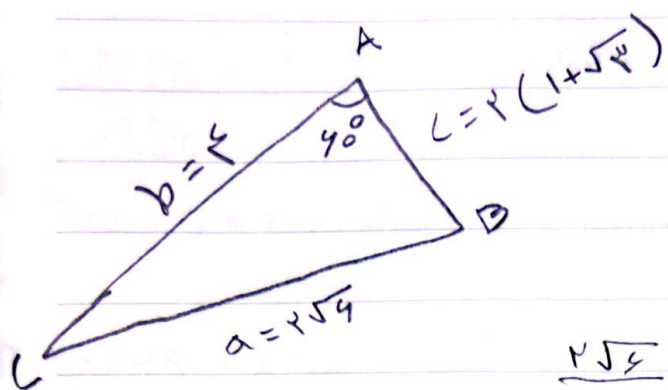
$$a^2 = b^2 + c^2 - 2(bc \cos A)$$

$$a^2 = 14^2 + 5^2 - 2(5 \times (1 + \sqrt{3}) \times \cos 60^\circ)$$

$$a^2 = 196 + 25 - 2(5 \times (1 + \sqrt{3}) \times \frac{1}{2})$$

$$a^2 = 221 - 5(1 + \sqrt{3}) = 221 - 5 - 5\sqrt{3} = 216 - 5\sqrt{3}$$

$$a = \sqrt{216 - 5\sqrt{3}} = 14$$



r

$\odot \sim \Delta$

$$\frac{a}{\sin A} = \frac{r}{\sin B}$$

(.)

$$\frac{\frac{r\sqrt{4}}{1}}{\frac{r\sqrt{4}}{r}} = \frac{r}{\sin B}$$

$$\frac{r\sqrt{4}}{\sqrt{4}} = \frac{r}{\sin B}$$

$$r\sqrt{4} = \frac{r}{\sin B}$$

$$\sin B = \frac{\sqrt{4}}{4} \Rightarrow \boxed{B = 40^\circ}$$

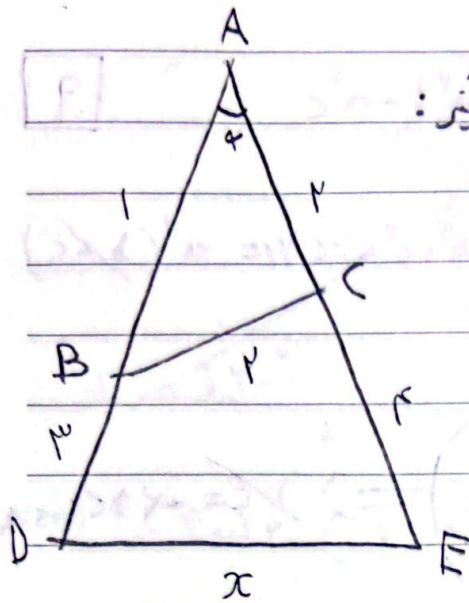
✓

$$\hat{A} + \hat{B} + \hat{C} = 180$$

$$40 + 40 + \hat{C} = 180$$

$$\boxed{\hat{C} = 100}$$

✓



در مثلث ABC قضیه کسینوس ها را برای کثیر:

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos 4$$

$$4 = 1 + 4 - (2 \times 1 \times 2 \times \cos 4)$$

$$4 = 5 - (4 \cos 4) \rightarrow 4 \cos 4 = 1 \quad \cos 4 = \frac{1}{4}$$

حالا در مثلث ADE قضیه کسینوس ها را داریم:

$$DE^2 = AD^2 + AE^2 - (2 \times AD \times AE \times \cos 4)$$

$$x^2 = 1^2 + 3^2 - (2 \times 1 \times 3 \times \frac{1}{4})$$

$$x^2 = 10 - 1.5 = 8.5 \quad x = \sqrt{8.5} = \frac{\sqrt{170}}{10}$$

$$BC^2 = AC^2 + AB^2 - (2 \times AC \times AB \times \cos 60^\circ)$$

الف) از قانون کسینوس ها استفاده می کنیم:

$$BC^2 = 2^2 + 9^2 - (2 \times 2 \times 9 \times \frac{1}{2})$$

$$BC^2 = 49 - 18 = 31 \quad BC = \sqrt{31} = \sqrt{31}$$

$$S_{ABC} = \frac{1}{2} \times AC \times AB \times \sin \hat{A} = \frac{1}{2} \times 2 \times 9 \times \frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{2}$$

$$\frac{b^2 + c^2 + 2bc - a^2}{2c(\cos A + 1)} = \frac{2bc \cos A + 2bc}{2c(\cos A + 1)}$$

$$a^2 = b^2 + c^2 - (2 \times b \times c \times \cos 4) \rightarrow b^2 + c^2 - a^2 = 2bc \cos 4$$

$$\frac{2bc(\cos A + 1)}{2c(\cos A + 1)} = \sqrt{31}$$

$$\frac{a^3 + b^3 - c^3}{a + b - c} = a^2 \quad \cancel{a^3} + \cancel{b^3} - \cancel{c^3} = \cancel{a^3} + a^2 b - a^2 c$$

9

$$b^3 - c^3 = a^2 b - a^2 c$$

$$(b - c)(b^2 + bc + c^2) = a^2(b - c)$$

(2)

بر اساس
قضیه کسینوس
تساوی داریم

این ها را برابر قرار می دهیم

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$-2 \cos A = 1$$

$$\cos A = -\frac{1}{2}$$

$$\rightarrow A = 120^\circ$$

10

$$\frac{1}{2} \times AB \times AC \times \sin 30^\circ = S_{ABC} = 2$$

$$\frac{1}{2} \times (3 - \cos \theta) \times (1 + 3 \cos \theta) \times \underbrace{\sin 30^\circ}_{\frac{1}{2}} = 2$$

$$(3 - \cos \theta)(1 + 3 \cos \theta) = 8$$

$$3 + 9 \cos \theta - \cos \theta - 3 \cos^2 \theta = 8$$

$$1 \cos \theta$$

$$3 \cos^2 \theta - 1 \cos \theta + 8 = 0$$

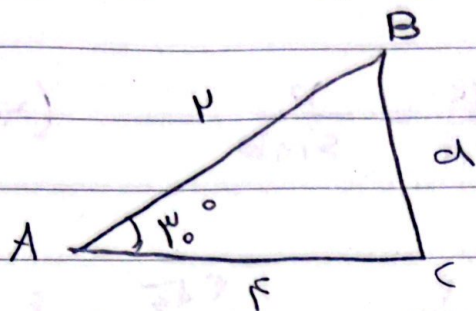
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(3)(8)}}{6} = \frac{1 \pm 2}{6}$$

$$\cos \theta < \frac{3}{2} \rightarrow \text{غیر قابل است زیرا } -1 \leq \cos \theta \leq 1$$

این جواب برای پذیر نیست

ادامه و ا ← دانستیم $\cos \theta = 1$ الفون دایر

حال برای پیدا کردن a از قانون
کسینوس ها استفاده می کنیم.



$$a^2 = AB^2 + AC^2 - (2 \times AB \times AC \times \cos \theta)$$

$$a^2 = 2^2 + 4^2 - (2 \times 2 \times 4 \times \cos 30^\circ) = 20 - 8\sqrt{3}$$