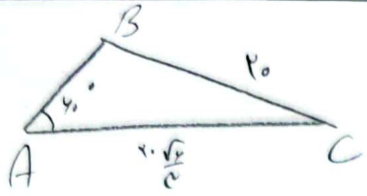
نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۱۲ ..... کلاس .....

۱



$\frac{BC}{\sin\hat{A}} = \frac{AC}{\sin\hat{B}} \rightarrow \frac{۲۰}{\frac{\sqrt{۶}}{۲}} = \frac{۲۰\sqrt{۶}}{۲\sin\hat{B}}$

$\sin\hat{B} = \frac{\sqrt{۲}}{۲} \times ۲۰ \times \frac{\sqrt{۶}}{۲۰} \times \frac{۱}{۲۰} = \frac{\sqrt{۲}}{۲}$

$\hat{B} \rightarrow ۴۵^\circ$

$\hat{B}+\hat{C} = ۱۸۰-۶۰ = ۱۲۰ \rightarrow \hat{A} = ۶۰^\circ$

$\hat{C} = ۱۲۰-۴۵^\circ = ۷۵^\circ$

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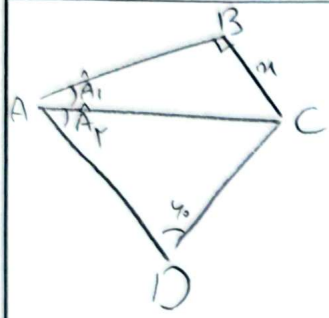


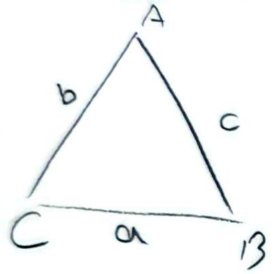
$BC = AC \times \sin\hat{A}_1 \rightarrow ۳ = AC \times \frac{\sqrt{۳}}{۲} \rightarrow AC = \frac{۶}{\sqrt{۳}}$

$\hat{A}_1 + \hat{C} + \hat{D} = ۱۸۰ \rightarrow \hat{A}_1 + ۷۵ + ۶۰ = ۱۸۰$

$\hat{A}_1 = ۴۵^\circ$

$\frac{CD}{\sin\hat{A}_1} = \frac{AC}{\sin\hat{D}} \rightarrow \frac{۲\sqrt{۲}}{\frac{\sqrt{۲}}{۲}} = \frac{\frac{۶}{\sqrt{۳}}}{\frac{\sqrt{۳}}{۲}} \rightarrow x = ۳$

۳



$\frac{c}{\sin\hat{C}} = \frac{b}{\sin\hat{B}} \rightarrow c = b\frac{\sin\hat{C}}{\sin\hat{B}}$

$c = (b^۲-۲)\frac{\sin\hat{C}}{\sin\hat{B}}$

$b = b^۲-۲ \rightarrow b^۲-b-۲ = ۰$

$(b-۲)(b+۱) = ۰ \xrightarrow{b>۰} b = ۲$ قبول

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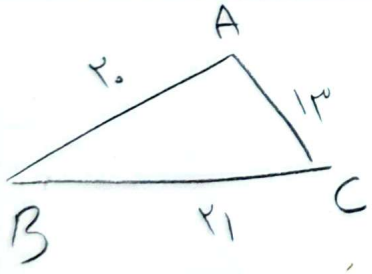
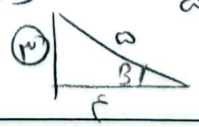


$AC^۲ = AB^۲ + CB^۲ - ۲AB \times CB \times \cos\hat{B}$

$۱۴۹ = ۴۰۰ + ۴۴۱ - ۲ \times ۲۰ \times ۲۱ \times \cos\hat{B}$

$\cos\hat{B} = ۰/۸ = \frac{۴}{۵}$ $\quad \sin^۲\hat{B} = ۱ - \cos^۲\hat{B}$



$\sin\hat{B} = \frac{۳}{۵} = ۰/۶$

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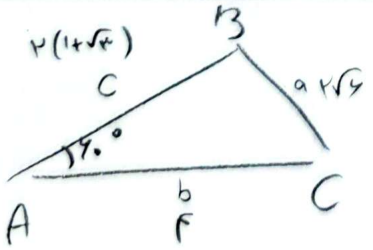

الف) $a^۲ = b^۲ + c^۲ - ۲bc\cos\hat{A}$

$a^۲ = ۱۶ + ۴(۴+۲\sqrt{۳}) - ۲ \times ۴ \times ۲(۱+\sqrt{۳}) \times \frac{۱}{۲}$

$\rightarrow a = \sqrt{۴۸} = ۴\sqrt{۳}$

ب) $\frac{c}{\sin\hat{C}} = \frac{b}{\sin\hat{B}} = \frac{a}{\sin\hat{A}} \rightarrow \frac{۲(۱+\sqrt{۳})}{\sin\hat{C}} = \frac{۴}{\sin\hat{B}} = \frac{۴\sqrt{۳}}{\frac{\sqrt{۳}}{۲}}$ ۵

$\rightarrow \sin\hat{B} = \frac{\sqrt{۲}}{۲} \rightarrow ۴۵^\circ$ $\quad \sin۷۵^\circ$

$\hat{C} = ۱۸۰-۶۰-۴۵ = ۷۵^\circ$

$BC^2 = AB^2 + AC^2 - 2AB \times AC \times \cos\alpha \rightarrow 3 = 1 + 4 - (2 \times 1 \times 2 \times \cos\alpha)$

$\rightarrow \cos\alpha = \frac{1}{4}$

$DE^2 = AD^2 + AE^2 - (2 \times AD \times AE \times \cos\alpha) \rightarrow 40 - \left(2 \times 4 \times 4 \times \frac{1}{4}\right)$

$\rightarrow x^2 - 12x + 32 = 0$ $\qquad DE = \sqrt{40} = 2\sqrt{10}$

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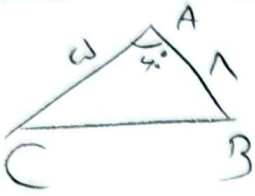

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos 60°$

$BC^2 = 64 + 25 - (2 \times 5 \times 8 \times \frac{1}{2}) = 49$

$BC = \sqrt{49} = 7$

$\therefore \rightarrow S = \frac{1}{2} \times AB \times AC \times \sin 60° \rightarrow \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

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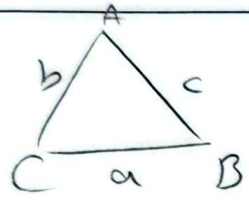

$\frac{(b+c)^2 - a}{bc(\cos\hat{A}+1)} \rightarrow \frac{2bc(\cos\hat{A}+1)}{bc(\cos\hat{A}+1)} = 2$

$a^2 = b^2 + c^2 - 2bc\cos\hat{A} \rightarrow (b+c)^2 - a^2$

$\Rightarrow b^2 + c^2 + 2bc - b^2 - c^2 + 2bc\cos\hat{A} = 2bc(1+\cos\hat{A})$

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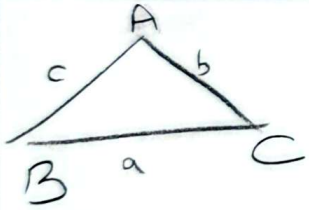

$\frac{a^3 + b^3 - c^3}{a+b-c} = a^2 \rightarrow a^3 + b^3 - c^3 = a^3 + a^2b - a^2c$

$\rightarrow (b-c)(b^2 + c^2 + bc) = a^2(b-c)$

$a^2 = b^2 + c^2 - 2bc\cos\hat{A} \rightarrow \cos\hat{A} = \frac{1}{2}$

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$S_{\hat{ABC}} = \frac{1}{2} AB \times AC \times \sin\hat{A} \rightarrow 2 = \frac{1}{2} \times \frac{1}{2} \times \frac{(3 - \cos\theta)(1 + 3\cos\theta)}{3 + 9\cos\theta - \cos\theta - 3\cos^2\theta}$

$\rightarrow -3\cos^2\theta + 8\cos\theta + 3 = 1 \rightarrow 3\cos^2\theta - 8\cos\theta + 5 = 0 \rightarrow \cos\theta = 1$

$3 - 3\cos\theta = 2$

$1 + 3\cos\theta = 4$

$\cos\theta = \frac{c}{a} = \frac{a}{3} \times \sqrt{\frac{5}{4}}$ $\qquad 0 < \cos\theta < 90°$

$a^2 = AB^2 + AC^2 - 2AB \times AC \times \cos A$

$\rightarrow a^2 = 4 + 14 - 12 \times 4 \times \frac{\sqrt{5}}{6} \rightarrow a^2 = 20 - 8\sqrt{5}$

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