دوازدهم (A) — تکلیف شماره ۳ — پریا پیشرونیا

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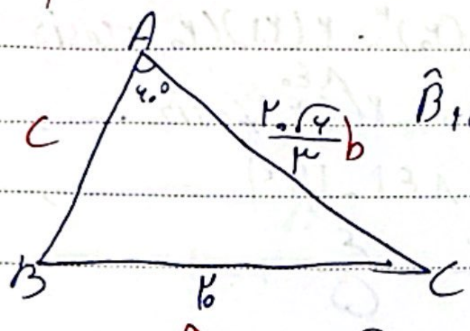


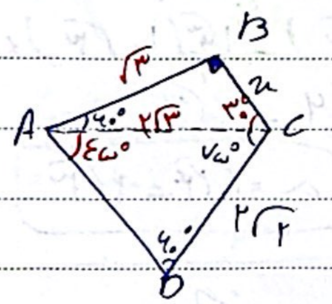
$\hat{B}+\hat{C}=120^\circ$, $A=60^\circ$, $\hat{B}=?$ $\hat{C}=?$

$\frac{a}{\sin A}=\frac{b}{\sin B} \rightarrow \frac{BC}{\frac{\sqrt{3}}{2}}=\frac{AC}{\sin B} \rightarrow$

$\rightarrow \frac{20}{\frac{\sqrt{3}}{2}}=\frac{\frac{20\sqrt{6}}{3}}{\sin B} \rightarrow \sin B=\frac{\sqrt{18}}{6}=\frac{3\sqrt{2}}{6}=\frac{\sqrt{2}}{2} \rightarrow \boxed{B=45^\circ}$

$\hat{B}+\hat{C}=120^\circ \rightarrow \hat{C}=120^\circ-45^\circ=75^\circ \rightarrow \boxed{C=75^\circ}$

۲-



$\frac{a}{\sin A}=\frac{d}{\sin D} \rightarrow \frac{2\sqrt{2}}{\sin 45}=\frac{AC}{\sin 60} \rightarrow$

$\rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}}=\frac{AC}{\frac{\sqrt{3}}{2}} \rightarrow AC=\frac{2\sqrt{6}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\sqrt{12} \rightarrow$

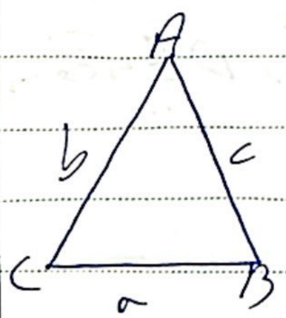
$\rightarrow AC=2\sqrt{3}$

در مثلث قائم‌الزاویه ضلع روبرو به زاویه 30° نصف وتر است

در نتیجه $AB=\frac{AC}{2} \rightarrow \underline{AB=\sqrt{3}}$

$AC^2=BC^2+AB^2 \rightarrow 12=x^2+3 \rightarrow x^2=9 \rightarrow \boxed{x=3}$

۳-



$(b^2-2)\frac{\sin\hat{C}}{\sin\hat{B}}=c$, $b=?$

$\frac{\sin\hat{C}}{\sin\hat{B}}=\frac{c}{b^2-2}$

$\frac{b}{\sin\hat{B}}=\frac{c}{\sin\hat{C}} \rightarrow \frac{\sin\hat{C}}{\sin\hat{B}}=\frac{c}{b}$

$\rightarrow \frac{c}{b^2-2}=\frac{c}{b} \rightarrow b^2-2=b \rightarrow b^2-b-2=0 \rightarrow b=-1$ (غ ق ق) و 2

$\boxed{b=2}$

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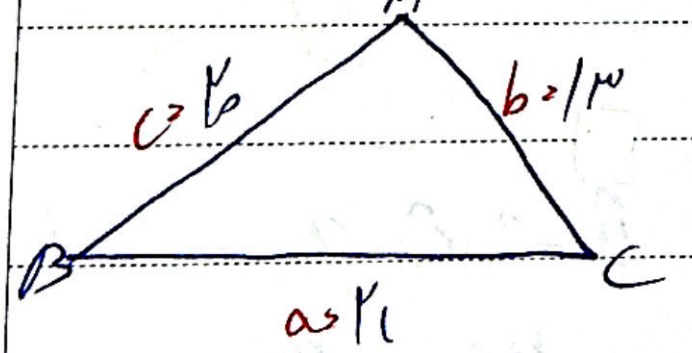


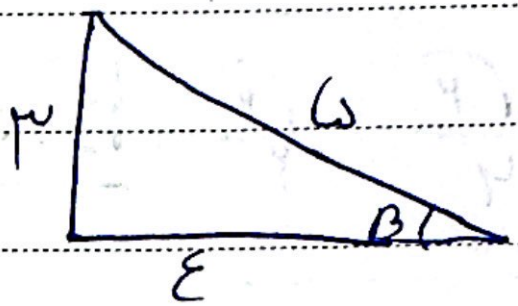
$\sin\hat{B} = ?$ حفظ کسینوس‌ها $\rightarrow b^2 = a^2 + c^2 - 2ac\cos\hat{B}$

$(13)^2 = (21)^2 + (20)^2 - 2(21)(20)\times\cos\hat{B}$

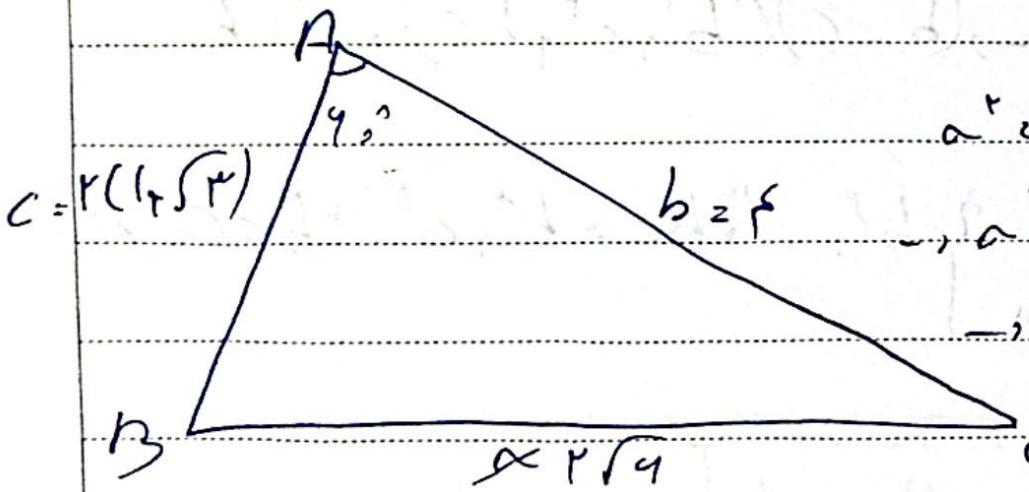
$169 = 441 + 400 - 840\cos\hat{B}$ $\rightarrow$

$\Rightarrow 840\cos\hat{B} = 841 - 169$ $\rightarrow$

$\rightarrow 840\cos\hat{B} = 672 \rightarrow \cos\hat{B} = \frac{672}{840} = \frac{\varepsilon}{\omega}$



$\sin\hat{B} = \frac{\text{مقابل}}{\text{وتر}} = \frac{۳}{\omega}$



۲(۴(۱+√۳)×$\frac{۱}{۲}$)

ب) $a^۲ = b^۲ + c^۲ - ۲bc\cos\hat{A}$ ← $a = ?$ (الف

← $a^۲ = ۱۶ + ۴(۴ + ۲\sqrt{۳}) - ۲(۴ + ۴\sqrt{۳})$ ←

← $a^۲ = ۱۶ + ۱۶ + ۸\sqrt{۳} - ۸ - ۸\sqrt{۳}$ ←

← $a^۲ = ۲۴$ ← $\boxed{a = \sqrt{۲۴} = ۲\sqrt{۶}}$

$\frac{b}{\sin B} = \frac{a}{\sin A}$ ← $\frac{۴}{\sin B} = \frac{۲\sqrt{۶}}{\frac{\sqrt{۳}}{۲}}$ ⇒ $\sin B = \frac{۲\sqrt{۳}}{۲\sqrt{۶}} \times \frac{\sqrt{۶}}{\sqrt{۶}} = \frac{\sqrt{۱۸}}{۶}$ ← (ب

← $\sin B = \frac{۳\sqrt{۲}}{۶} = \frac{\sqrt{۲}}{۲}$ ← $\boxed{B = ۴۵^\circ}$

$\hat{A} + \hat{B} + \hat{C} = ۱۸۰^\circ$ ← $۹۰ + ۴۵ + \hat{C} = ۱۸۰$ ← $\boxed{\hat{C} = ۴۵^\circ}$

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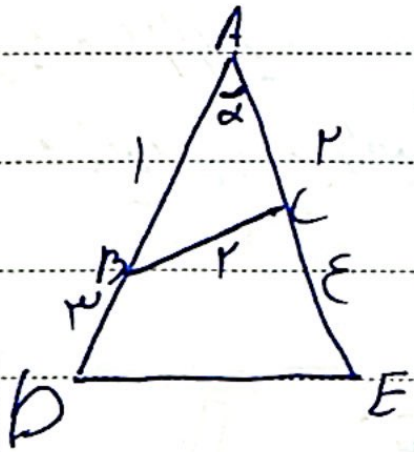

$DE = ?$ $\quad a^2 = b^2 + c^2 - 2bc\cos\hat{A} \longrightarrow$

$\longrightarrow BC^2 = AC^2 + AB^2 - 2(AC \times AB \times \cos\alpha) \longrightarrow$

$\longrightarrow 4 = 4 + 1 - 2(\underbrace{2 \times 1 \times \cos\alpha}_{2\cos\alpha}) \Longrightarrow$

$\longrightarrow 4 = 5 - 4\cos\alpha \to 4\cos\alpha = 1 \to \cos\alpha = \frac{1}{4}$

ADE مثلث $\longrightarrow DE^2 = AD^2 + AE^2 - 2(\underbrace{\overset{6}{AE} \times \overset{4}{AD} \times \cos\alpha}_{24\cos\alpha}) \longrightarrow$

$\longrightarrow DE^2 = 16 + 36 - 48\underset{\frac{1}{4}}{\cos\alpha} \longrightarrow DE^2 = 40 \longrightarrow DE = \sqrt{40} \longrightarrow$

$\longrightarrow DE = 2\sqrt{10}$

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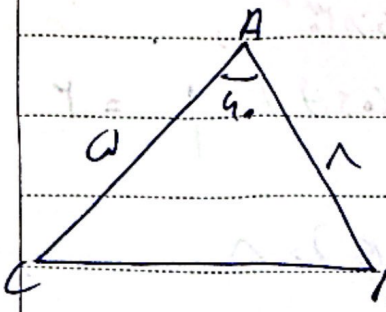

۷- $\frac{8\times 5}{2} = 20$

$BC = ?$ ، $BC^2 = AB^2 + AC^2 - 2(AB \times AC \times \cos 60)$ (قانون کسینوس‌ها)

$\Rightarrow BC^2 = 64 + 25 - 2(20) \Rightarrow BC^2 = 49 \xrightarrow{\sqrt{}} \boxed{BC = 7}$

$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin 60^\circ = \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt{3}}{2} = \boxed{10\sqrt{3}}$ (ب

$$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} = \frac{b^2+c^2+2bc-a^2}{bc(\cos\hat{A}+1)} \xrightarrow{b^2+c^2-a^2 = 2bc\cos\hat{A}} \frac{2bc\cos\hat{A}+2bc}{bc(\cos\hat{A}+1)} \Rightarrow$$

$$\Rightarrow \frac{2bc(\cos\hat{A}+1)}{bc(\cos\hat{A}+1)} = \boxed{2}$$

$$\frac{a^3+b^3-c^3}{a+b-c} = a^2 \Rightarrow a^3+b^3-c^3 = a^3+a^2b-a^2c \quad (9$$

$$\Rightarrow b^3 - c^3 = a^2(b-c) \Rightarrow (b-c)(b^2+c^2+bc) = a^2(b-c)$$

$$\Rightarrow a^2 = b^2+c^2+bc \quad \left\{ b^2+c^2+bc = b^2+c^2-2bc\cos\hat{A} \Rightarrow \right.$$

قضیه کسینوس‌ها $\Rightarrow a^2 = b^2+c^2-2bc\cos\hat{A}$ $\left.\right\}$

$$\Rightarrow bc = -2bc\cos\hat{A} \Rightarrow \boxed{\cos\hat{A} = -\frac{1}{2}}$$

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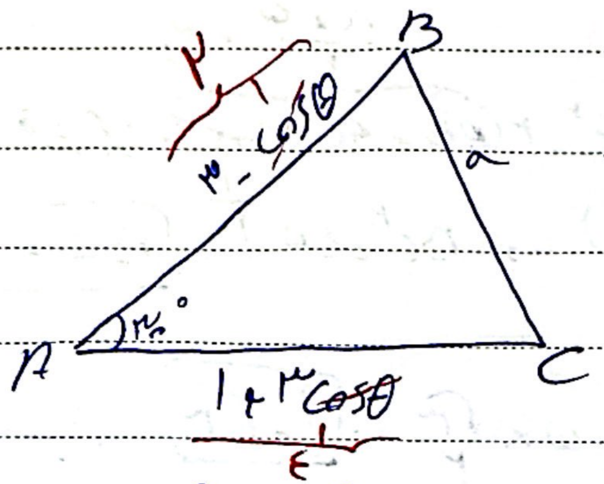

$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin 30^\circ \to \frac{1}{2}(3 - \cos\theta)(1 + 3\cos\theta) \times \frac{1}{2} = 2$

$(3 - \cos\theta)(1 + 3\cos\theta) = 8 \to$

$\to 3 + 9\cos\theta - \cos\theta - 3\cos^2\theta = 8 \to 3\cos^2\theta - 8\cos\theta + 5 = 0 \to$

$\to \cos\theta = 1 , \frac{5}{3}$ (غ‌ق‌ق) $\to \cos\theta = 1$

← $\frac{5}{3}$ از ۱ بزرگتر است و $-1 \le \cos\theta \le 1$

قضیه کسینوس‌ها $\to a^2 = AB^2 + AC^2 - 2\underbrace{(AB \times AC \times \cos 30^\circ)}_{4\sqrt{3}} \to$ $\left(\cos 30^\circ = \frac{\sqrt{3}}{2}\right)$

$\to a^2 = 4 + 16 - 8\sqrt{3} \to \boxed{a^2 = 20 - 8\sqrt{3}}$