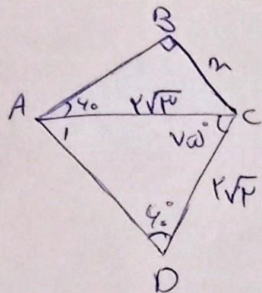


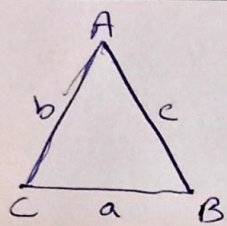
$$\begin{aligned}\hat{A} + \hat{B} + \hat{C} &= 180^\circ \\ \hat{B} + \hat{C} &= 140^\circ \\ \hat{A} &= 40^\circ\end{aligned}$$

$$\frac{BC}{\sin \hat{A}} = \frac{AC}{\sin \hat{B}} \Rightarrow \frac{10}{\sin 40^\circ} = \frac{10\sqrt{4}}{\sin \hat{B}} \Rightarrow \frac{10}{\sin 40^\circ} = \frac{20}{\sin \hat{B}} \Rightarrow \sin \hat{B} = \frac{2 \sin 40^\circ}{1} \Rightarrow \hat{B} = 40^\circ, \hat{C} = 100^\circ$$



$$\hat{A}_1 = 180 - 40 - 100 = 40^\circ$$

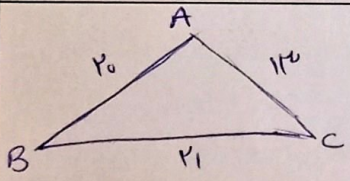
$$\frac{CD}{\sin 40^\circ} = \frac{AC}{\sin 40^\circ} \Rightarrow \frac{CD}{\sin 40^\circ} = \frac{10\sqrt{4}}{\sin 40^\circ} \Rightarrow CD = 10\sqrt{4}$$



$$\frac{(b^2 - r)(\sin \hat{C})}{\sin \hat{B}} = \frac{c}{1} \Rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b^2 - r} \Rightarrow \frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b^2 - r}$$

$$\frac{c}{\sin \hat{C}} = \frac{b^2 - r}{\sin \hat{B}} \Rightarrow \frac{c}{\sin \hat{C}} = \frac{b}{\sin \hat{B}} \Rightarrow b^2 - r = b$$

$$b^2 - b - r = 0 \quad (b-1)(b+1) = 0 \Rightarrow b = -1 \text{ (رد)} \Rightarrow b = 1 \text{ (قبول)}$$



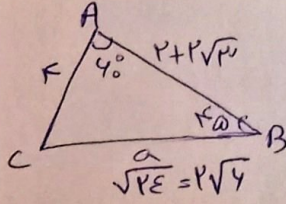
$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos \hat{B}$$

$$14^2 = 10^2 + 11^2 - 2(10)(11)\cos \hat{B}$$

$$\cos \hat{B} = \frac{44}{110} = \frac{2}{5}$$

$$\sin^2 \hat{B} + \cos^2 \hat{B} = 1 \Rightarrow \frac{16}{25} + \sin^2 \hat{B} = 1 \Rightarrow \sin^2 \hat{B} = \frac{9}{25} \Rightarrow \sin \hat{B} = \frac{3}{5}$$

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$$a^2 = 14^2 + (10+11)^2 - 2(14)(21)\cos \hat{B} \Rightarrow$$

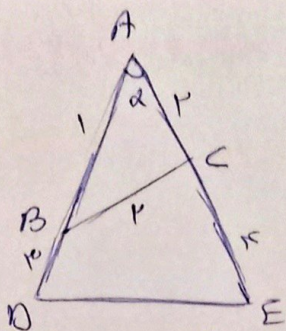
$$a^2 = 14^2 + 441 - 2(14)(21)\cos \hat{B} = 25 \Rightarrow a = 5$$

$$\frac{a}{\sin 40^\circ} = \frac{10}{\sin \hat{B}} \Rightarrow \frac{5}{\sin 40^\circ} = \frac{10}{\sin \hat{B}} \Rightarrow \sin \hat{B} = \frac{2 \sin 40^\circ}{1} \Rightarrow \hat{B} = 40^\circ$$

$$\hat{C} = 180 - 40 - 40 = 100^\circ$$

(الف)

(ب)



$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\alpha$$

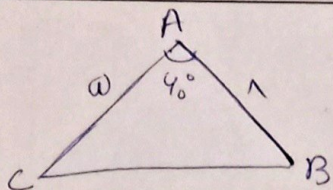
$$4 = 1 + 4 - 2 \times 1 \times 2 \cos\alpha \Rightarrow \cos\alpha = \frac{1}{2} \checkmark$$

$$DE^2 = AD^2 + AE^2 - 2(AD)(AE)\cos\alpha$$

$$DE^2 = \underbrace{1^2 + 2^2}_{=5} - 2 \times 1 \times 2 \times \frac{1}{2} \Rightarrow DE^2 = 4 \checkmark$$

$$DE = \sqrt{4} = 2 \checkmark$$

(Y)



$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 40^\circ$$

$$BC^2 = \underbrace{10^2 + 10^2}_{=200} - 2 \times 10 \times 10 \times \frac{1}{2} = 100 \Rightarrow BC = 10 \checkmark$$

$$S = \frac{1}{2} \times AB \times AC \times \sin 40^\circ =$$

$$\frac{1}{2} \times 10 \times 10 \times \frac{\sqrt{10}}{2} = 10\sqrt{10} \checkmark$$

(Y)

$$\frac{(b+c)^2 - a^2}{bc \cos \hat{A} + bc} = \frac{b^2 + c^2 + 2bc - a^2}{bc \cos \hat{A} + bc} = \frac{b^2 + c^2 + 2bc - a^2 - 2bc \cos \hat{A} - 2bc}{bc \cos \hat{A} + bc}$$

$$\xrightarrow{\text{Simplification}} \frac{b^2 + c^2 - 2bc \cos \hat{A} + 2bc - 2bc - a^2}{bc \cos \hat{A} + bc}$$

$$a^2 = b^2 + c^2 - 2bc \cos \hat{A}$$

$$\frac{a^2 - a^2}{bc \cos \hat{A} + bc} = 0$$

$$\frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos \hat{A}}{bc (\cos \hat{A} + 1)} = \frac{2bc (\cos \hat{A} + 1)}{bc (\cos \hat{A} + 1)} = 2$$

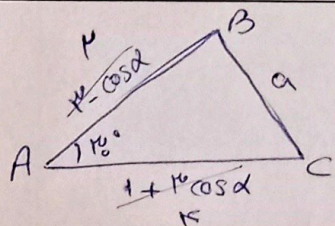
$$\frac{a^2 + b^2 - c^2}{a + b - c} = a^2 \Rightarrow a^2 + b^2 - c^2 = a^2 + b a^2 - c a^2$$

$$(b-c)(b^2 + c^2 + bc) = a^2(b-c)$$

$$a^2 = b^2 + c^2 + bc \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow bc = -2bc \cos A \Rightarrow \cos A = -\frac{1}{2} \rightarrow \hat{A} = 120^\circ \checkmark$$

(Y)



$$S = \frac{1}{2} \times \sin 120^\circ \times (1 - \cos \alpha)(1 + \cos \alpha) = 1 \quad \cos \alpha = \frac{1}{2}$$

$$\Rightarrow (1 - \frac{1}{2})(1 + \frac{1}{2}) = 1 \quad -\frac{1}{2} + \frac{1}{2} - 1 = 0 \quad \frac{1}{2} = 1 \quad \frac{1}{2} = 1$$

(Y)

$$a^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 120^\circ$$

$$a^2 = 1 + 1 - 2 \times 1 \times 1 \times (-\frac{1}{2}) = 2 \Rightarrow a = \sqrt{2} \checkmark$$

$$-1 \leq \cos \alpha \leq 1$$