

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\hat{A} = 120^\circ + 40^\circ = 160^\circ$$

$$\frac{10}{\frac{\sqrt{3}}{2}} = \frac{10\sqrt{4}}{\cancel{\sqrt{3}} \sin B}$$

$$\frac{10\sqrt{2}}{4} = \frac{10}{\sqrt{3}} \sin B \rightarrow \hat{B} = 40^\circ \checkmark$$

$$\hat{C} = 120^\circ - 40^\circ = 80^\circ \checkmark$$

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$$A_1 = 120^\circ - (120^\circ + 40^\circ) = 40^\circ$$

$$\frac{10\sqrt{2}}{\frac{\sqrt{3}}{2}} = \frac{10\sqrt{2}}{\frac{\sqrt{3}}{2}} \rightarrow \frac{a}{\sin \hat{A}} = \frac{d}{\sin \hat{B}}$$

$$x = d \cdot \sin 40^\circ = 10\sqrt{2} \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3} \checkmark$$

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$$\frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow c = \frac{\sin C}{\sin B} (b) = \frac{\sin C}{\sin B} (b^2 - 1)$$

$$b^2 - b - 1 = 0 \rightarrow \begin{cases} b = 1 \\ b = -1 \end{cases} \checkmark$$

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$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$14 = 16 + 16 - 2(4)(4) \cos B$$

$$\cos B = \frac{1}{2} \checkmark \rightarrow \sin \frac{120^\circ}{2} = 0.5 \checkmark$$

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$$a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a^2 = 14 + 14 + 1\sqrt{3} - (14 + 14 \cos 120^\circ)$$

$$12 - 1 = 2\sqrt{3} \rightarrow a = 2\sqrt{3} \checkmark$$

~~$$14 = 16 + 16 - 2(4)(4) \cos A$$~~

$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sin B} \rightarrow \hat{B} = 30^\circ \checkmark \text{ و } \hat{C} = 30^\circ \checkmark$$

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$5^2 = 1^2 + 4^2 - 2(1)(4) \cos A \rightarrow \cos A = \frac{1}{4} \checkmark$$

$$a^2 = 1^2 + 4^2 - 2(1)(4) \left(\frac{1}{4}\right) = 1^2 + 4^2 - 2 = 16 \quad \textcircled{y}$$

$$DE = a = \sqrt{16} = 4 \checkmark$$

در $\triangle ABC$

و $\triangle ADE$

$$BC^2 = 1^2 + 4^2 - 2(1)(4) \cos \frac{1}{4} = 1^2 + 4^2 - 2 = 16 \rightarrow BC = 4 \checkmark$$

الف

$$S = b \cdot c \cdot \frac{1}{2} \sin A \rightarrow 1 \times 4 \times \frac{1}{2} \times \frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{2} \checkmark$$

ب. v

$$a^2 = b^2 + c^2 - 2bc \cos A^{\wedge} \quad \textcircled{y}$$

$$\frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos A}{2bc (\cos A + 1)} = \frac{2bc \cos A + 2bc}{2bc (\cos A + 1)} = \frac{2bc (\cos A + 1)}{2bc (\cos A + 1)} = 1$$

$$a^2 + b^2 - c^2 = a^2 + a^2 b - a^2 c$$

$$(b-c)(b^2 + c^2 + bc) = a^2(b-c) \rightarrow \left. \begin{aligned} a^2 &= b^2 + c^2 + bc \\ a^2 &= b^2 + c^2 - 2bc \cos A \end{aligned} \right\} \quad \textcircled{y}$$

$$\cos A = -\frac{1}{4} \rightarrow A^{\wedge} = 104^{\circ}$$

$$S = \frac{1}{2} \times \frac{\sqrt{15}}{4} \times (1 - \cos \theta) (1 + 4) = 1 \rightarrow 1 - \cos \theta + 4 \cos \theta = 1$$

$$1 - \cos \theta + 4 \cos \theta = 1 \rightarrow \cos \theta = 1 \rightarrow C = 0^\circ \text{ و } b = 4 \quad \textcircled{y}$$

$$a^2 = 1^2 + 4^2 - 2(1)(4) \cos A = 1^2 + 4^2 - 1 \times 4 = 16 - 4 = 12 \rightarrow a = \sqrt{12} = 2\sqrt{3} \checkmark$$