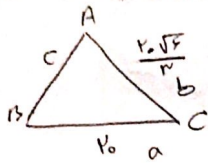


١) $\triangle ABC \rightarrow BC = 10, \hat{B} + \hat{C} = 110^\circ, AC = \frac{10\sqrt{6}}{3} \rightarrow \hat{C} = ? \hat{B} = ?$



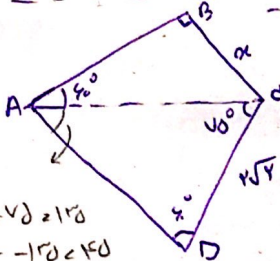
مبدأ جيب

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{10}{\sin 90^\circ} = \frac{\frac{10\sqrt{6}}{3}}{\sin \hat{B}} \rightarrow \sin \hat{B} = \frac{\frac{10\sqrt{6}}{3}}{10} = \frac{\sqrt{6}}{3}$$

$$\hat{B} + \hat{C} = 110^\circ \rightarrow \hat{A} = 180^\circ - 110^\circ = 70^\circ$$

$$\rightarrow \sin \hat{B} = \frac{\sqrt{6}}{3} = \frac{10\sqrt{6}}{3} \cdot \frac{1}{10} = \frac{\sqrt{6}}{3} \rightarrow \hat{B} = 40^\circ \rightarrow C = 110^\circ - 40^\circ = 70^\circ$$

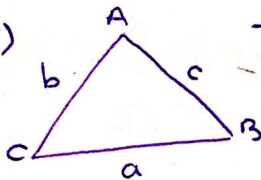
٢)



$$\rightarrow \hat{C} = ? \rightarrow \frac{BC}{\sin \hat{C}} = \frac{AC}{\sin 40^\circ} \rightarrow BC = \frac{10\sqrt{6} \times \frac{\sqrt{6}}{3}}{1} = 20$$

$$\frac{AC}{\sin 40^\circ} = \frac{10\sqrt{6}}{\sin 40^\circ} \rightarrow AC = \frac{10\sqrt{6} \times \frac{\sqrt{6}}{3}}{\frac{\sqrt{6}}{3}} = 20\sqrt{6}$$

٣)

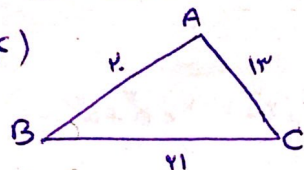


$$\rightarrow (b^2 - 1) \frac{\sin \hat{C}}{\sin \hat{B}} = c \rightarrow b = ?$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b} \rightarrow (b^2 - 1) \frac{\sin \hat{C}}{\sin \hat{B}} = c \rightarrow$$

$$\frac{b^2 - 1}{b} = 1 \rightarrow b^2 - b - 1 = 0 \rightarrow (b - 2)(b + 1) = 0 \rightarrow b = 2$$

٤)



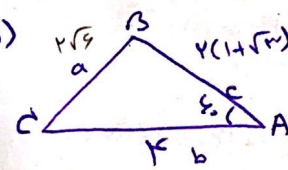
$$\rightarrow \sin \hat{B} = ?$$

$$\rightarrow \text{مبدأ جيب} \rightarrow AC^2 = AB^2 + BC^2 - 2(AB)(BC) \cos \hat{B} \rightarrow$$

$$10^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \cos \hat{B} \rightarrow 100 = 200 - 200 \cos \hat{B} \rightarrow 200 \cos \hat{B} = 100 \rightarrow \cos \hat{B} = \frac{1}{2} \rightarrow \hat{B} = 60^\circ$$

$$\cos \hat{B} = \frac{1}{2} = \frac{10}{20} \rightarrow \hat{B} = 60^\circ$$

٥)



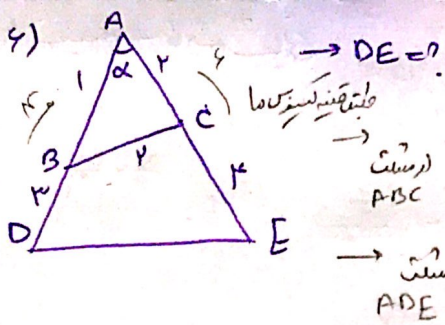
$$\text{الف) } a = ? (BC) \rightarrow$$

$$a^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \cos 60^\circ = 100 + 100 - 100 = 100 \rightarrow a = 10$$

ب) $\hat{B} = ? \hat{C} = ?$

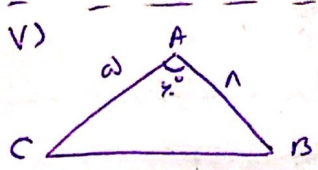
$$\rightarrow \text{مبدأ جيب} \rightarrow \frac{10\sqrt{6}}{3} = \frac{10}{\sin \hat{B}} \rightarrow \sin \hat{B} = \frac{10}{10\sqrt{6}/3} = \frac{3}{\sqrt{6}} = \frac{\sqrt{6}}{2} \rightarrow \hat{B} = 60^\circ$$

$$\rightarrow \sin \hat{B} = \frac{\sqrt{6}}{2} \rightarrow \hat{B} = 60^\circ \rightarrow 180^\circ - 60^\circ - 60^\circ = 60^\circ = \hat{C}$$



$\rightarrow$ $\vec{BC} \rightarrow r = r + 1 - r \cos \alpha \rightarrow r \cos \alpha = 1 \rightarrow \cos \alpha = \frac{1}{r}$ ✓

$\rightarrow$ $\vec{ADE} \rightarrow DE^2 = 1^2 + r^2 - \frac{r \times 1 \times \frac{1}{r}}{1r} = r^2 \rightarrow \boxed{DE = r\sqrt{1 - \frac{1}{r^2}}}$ ✓



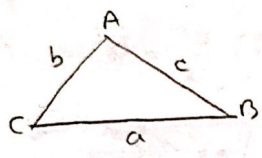
ان $BC = ?$

$\rightarrow BC^2 = 1^2 + 1^2 - 1 \times 1 \times \frac{1}{r} = 2 - \frac{1}{r} \Rightarrow \boxed{BC = \sqrt{2 - \frac{1}{r}}}$ ✓

ب) $S_{ABC} = ? \rightarrow$

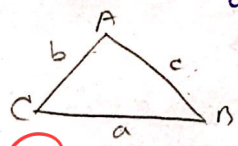
$S_{ABC} = \frac{1}{2} ab \sin \theta \Rightarrow S = \frac{1}{2} \times 1 \times 1 \times \frac{\sqrt{r}}{r} = \boxed{\frac{1}{2} \sqrt{r}}$ ✓

6) $\vec{ABC} \rightarrow \frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} = ?$



$\rightarrow a^2 = b^2 + c^2 - 2bc \cos \hat{A} \rightarrow \frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} = \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} = \frac{2bc(1 + \cos \hat{A})}{bc(1 + \cos \hat{A})} = \boxed{2}$ ✓

7) $\vec{ABC} \rightarrow \frac{a^2 + b^2 - c^2}{a+b-c} = a^2 \rightarrow \cos \hat{A} = ?$



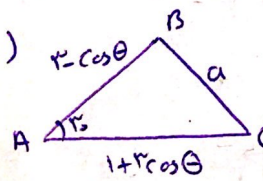
$\frac{a^2 + (b-c)(b^2 + bc + c^2)}{a+b-c} = a^2 \rightarrow a^2 + (b-c)(b^2 + bc + c^2) = a^2 + a^2(b-c)$

$\rightarrow b^2 + c^2 + bc = a^2$

$\rightarrow b^2 + c^2 - 2bc \cos \hat{A} = a^2 \rightarrow bc = \frac{1}{2} bc \cos \hat{A} \rightarrow \cos \hat{A} = \frac{1}{r}$ ✓

$\rightarrow \boxed{A = 120^\circ}$

8) $\vec{ABC} = r \rightarrow a^2 = ?$



$a^2 = (r \cos \theta)^2 + (1 + r \cos \theta)^2 - r(r \cos \theta)(1 + r \cos \theta) \times \frac{\sqrt{r}}{r}$

$\rightarrow S = \frac{1}{2} \times AB \times AC \times \sin \theta = \frac{1}{2} \times (r \cos \theta)(1 + r \cos \theta) \times \frac{1}{r} = r \rightarrow (r \cos \theta)(1 + r \cos \theta) = 1$

$\rightarrow 4r^2 \cos^2 \theta + r - r \cos \theta - r^2 \cos^2 \theta = 1 \rightarrow r^2 \cos^2 \theta - r \cos \theta + r = 1$

$\rightarrow r^2 \cos^2 \theta - r \cos \theta + 1 = 0 \rightarrow \cos \theta = \frac{r}{2} \times \frac{1}{r} = \frac{1}{2}$

$\rightarrow a^2 = (r-1)^2 + (1+r)^2 - r(1) \times \frac{\sqrt{r}}{r} = r + 1 - \sqrt{r} = \boxed{r + 1 - \sqrt{r}}$ ✓