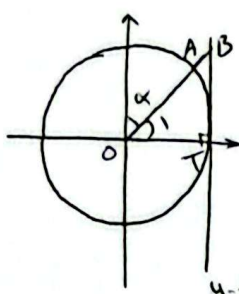
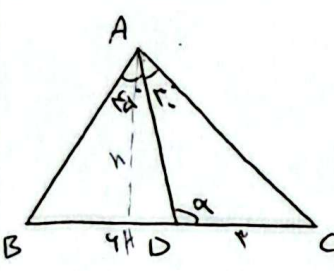
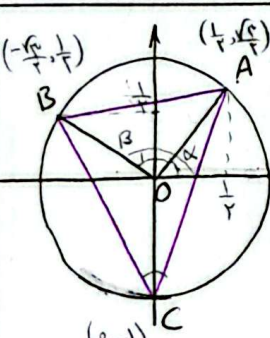


$\hat{\alpha} + \hat{\alpha}_1 = \pi \Rightarrow \sin \alpha = \cos \hat{\alpha}_1$

 $\cos \hat{\alpha}_1 = \frac{OT}{OB} = \frac{OT}{OA + AB} \xrightarrow{\text{مماسی}} \cos \hat{\alpha}_1 = \frac{1}{1 + AB}$
 $\sin \alpha = \frac{1}{1 + AB} \Rightarrow 1 + AB = \frac{1}{\sin \alpha}$
 $AB = \frac{1 - \sin \alpha}{\sin \alpha}$

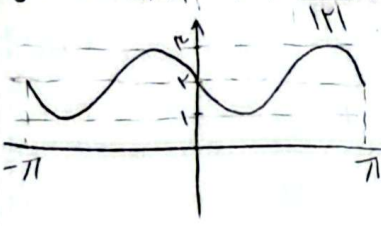

 $\Rightarrow \frac{S_{ABD}}{S_{ADC}} = \frac{\frac{1}{2} \times 4 \times y}{\frac{1}{2} \times r \times y} = \frac{AB \times AD \times \frac{1}{2} \times \sin \alpha}{AC \times AD \times \frac{1}{2} \times \sin \alpha}$
 $\frac{AB}{AC} = \frac{4}{r} \times \frac{1}{\frac{r}{4}} = \sqrt{2}$


 $\cos \alpha = \frac{1}{r} \Rightarrow \alpha = 2^\circ$
 $\sin \beta = \frac{1}{r} \Rightarrow \beta = 14^\circ$
 $\Rightarrow \hat{C} = \frac{1}{r} \hat{O}_1 \Rightarrow \hat{C} = 4^\circ$
 $AC = \sqrt{\left(\frac{1}{r} - 0\right)^2 + \left(\frac{\sqrt{r}}{r} + 1\right)^2} = \frac{\sqrt{1 + r}}{r}$
 $BC = \sqrt{\left(-\frac{\sqrt{r}}{r} - 0\right)^2 + \left(\frac{1}{r} - 1\right)^2} = \frac{\sqrt{r}}{r} = \sqrt{r}$
 $S_{ABC} = AC \times BC \times \sin \hat{C} \times \frac{1}{2} \Rightarrow \frac{\sqrt{1 + r}}{r} \times \sqrt{r} \times \frac{\sqrt{r}}{r} \times \frac{1}{2} = \frac{\sqrt{1 + r}}{2r}$
 $AB = \sqrt{\left(\frac{1}{r} + \frac{\sqrt{r}}{r}\right)^2 + \left(\frac{\sqrt{r}}{r} - \frac{1}{r}\right)^2} = \sqrt{r} \Rightarrow \frac{1}{2} \times \sqrt{r} \times \sqrt{r} + \frac{\sqrt{1 + r}}{2r} = \sqrt{r}$

$S_{ABC} = \frac{1}{2} AB \times AC \times \sin \hat{A} \Rightarrow \frac{1}{2} \times \sqrt{r} \times \frac{\sqrt{1 + r}}{r} \times \frac{1}{r} = \frac{1}{2} \times \sqrt{r} \times \frac{\sqrt{1 + r}}{r} \times \frac{1}{r} = \sqrt{r}$

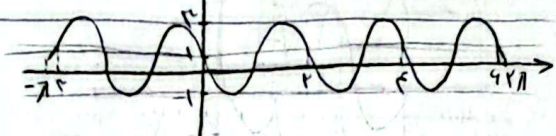
$\frac{r \sin^2 \alpha - r \sin^2 \alpha (\sin^2 \alpha - 1)}{\sin^2 \alpha - \cos^2 \alpha} \xrightarrow{\frac{1 - \cos \alpha}{\cos \alpha}} \frac{\cos^2 \alpha - \cos^2 \alpha + \cos^2 \alpha - \sin^2 \alpha + \sin^2 \alpha \cos^2 \alpha}{\cos^2 \alpha (\cos^2 \alpha - 1) (\sin^2 \alpha + \cos^2 \alpha) (\cos^2 \alpha - \sin^2 \alpha)}$
 $\Rightarrow -\cos^2 \alpha \sin^2 \alpha + \cos^2 \alpha + \sin^2 \alpha \cos^2 \alpha = \cos^2 \alpha = 1$
 $\hookrightarrow \sin^2 \alpha = \sqrt{1 - 1} = 0$

$\text{C61) } y = -r \sin x \cos x + r$
 $y = -\sin x + r \Rightarrow T = \frac{r\pi}{|r|} = \pi \checkmark$

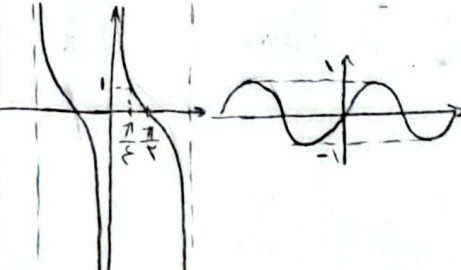
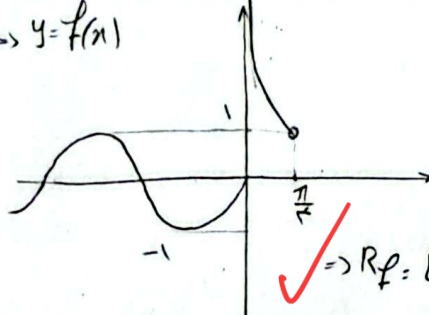


(r)

$\Rightarrow y = r \cos\left(x + \frac{1}{r}\right)\pi + 1$
 $y = r \cos \pi x + \frac{\pi}{r} + 1 = -r \sin \pi x + 1$
 $\hookrightarrow T = \frac{r\pi}{|r|} = r \checkmark$



$y = \cot x$ $y = \sin x$ $\Rightarrow y = f(x)$

(r)

$\checkmark \Rightarrow R_f = [-1, +\infty)$

$f(x) = a + b \sin(cx) \cos(cx) \cos(rx) = a + \frac{b}{r} \sin(rx) \cos(rx) = a + \frac{b}{r} \sin(rx)$
 $T = \frac{r\pi}{|r|} \Rightarrow \frac{r\pi}{|r|} \cdot \frac{r\pi}{|r|} \rightarrow |r| = a \Rightarrow c = \frac{a}{r} \checkmark$
 $\max_{0 \leq x < 2\pi} = r \cdot \frac{a}{r} \rightarrow \max_{0 \leq x < 2\pi} = r \Rightarrow |b| = r \Rightarrow b = -r$
 $\frac{bc}{a} = \frac{-r \cdot \frac{a}{r}}{r} = \boxed{-\frac{a}{r}}$
 $\frac{bc}{a} = a$
 $\Rightarrow a = r$
 $\Rightarrow a + b = r \Rightarrow a = r$

(1,0)

$f(x) = a \cos bx + c$
 $T = \frac{r\pi}{|b|} \Rightarrow \pi = \frac{r\pi}{|b|} \Rightarrow |b| = r \Rightarrow b = r$
 $|a| + c = r$
 $-|a| + c = -r$
 $\Rightarrow \begin{cases} r = -r \\ c = -1 \end{cases}$
 $|a| - 1 = r$
 $|a| = r$
 $a = -r$
 $(a < 0)$

$\Rightarrow f(x) = -r \cos rx - 1$
 $-r \cos rx - 1 = 0$
 $r \cos rx = -1$
 $\cos rx = -\frac{1}{r} \Rightarrow 1 + \tan^2 rx = \frac{1}{\cos^2 rx} = \frac{1}{\frac{1}{r^2}} = r^2$
 $\tan^2 rx = r^2 - 1 \Rightarrow |\tan rx| = \sqrt{r^2 - 1}$

(r)

$T = r \Rightarrow f(x) = f(x + r\pi) \Rightarrow f(-r\pi, a) = f(1, a - r\pi) = f(1, a - r(r)) = f(1, a)$
 $\Rightarrow f(x) = -\frac{1}{r}x + 1 \Rightarrow f\left(\frac{r}{r}\right) = -\frac{1}{r} \cdot \frac{r}{r} + 1 = \boxed{\frac{1}{r}} \checkmark$
 $0 \leq x < r$

(r)