

$$A(\cos(\frac{\pi}{4}-\alpha), \sin(\frac{\pi}{4}-\alpha)) \Rightarrow A(\sin\alpha, \cos\alpha)$$

$$B(1, \tan(\frac{\pi}{4}-\alpha)) \Rightarrow B(1, \cot\alpha)$$

$$\overline{AB} = \sqrt{(\sin\alpha-1)^2 + (\cos\alpha-\cot\alpha)^2} = \sqrt{\sin^2\alpha - 2\sin\alpha + 1 + \frac{(\cos\alpha - \frac{\cos\alpha}{\sin\alpha})^2}{\sin^2\alpha}} = \sqrt{\sin^2\alpha - 2\sin\alpha + 1 + \frac{\cos^2\alpha(1-\frac{1}{\sin\alpha})^2}{\sin^2\alpha}}$$

$$\Rightarrow \overline{AB} = \sqrt{(\sin\alpha-1)^2 (1 + \frac{\cos^2\alpha}{\sin^2\alpha})} = \frac{\sin\alpha-1}{\sin\alpha} \cdot \frac{1-\sin\alpha}{\sin\alpha} = \frac{(1-\sin\alpha)^2}{\sin\alpha}$$

$$\text{در مثل ADC: } \frac{w}{\sin\alpha} = \frac{AC}{\sin\alpha} \Rightarrow AC = w \sin\alpha$$

$$\Rightarrow \frac{AB}{AC} = \frac{\frac{(1-\sin\alpha)^2}{\sin\alpha}}{w \sin\alpha} = \frac{1-\sin\alpha}{w \sin^2\alpha}$$

$$\text{ADB مثل: } \frac{y}{\sin\alpha} = \frac{AB}{\sin\alpha} \Rightarrow AB = y \sin\alpha$$

$$A(\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{2}}), B(-\frac{\sqrt{2}}{\sqrt{2}}, \frac{1}{\sqrt{2}}), C(0, -1)$$

- با توجه به داده های سوال:

$$S_{ABC} = S_{ABO} + S_{BOC} + S_{AOC} = \frac{1}{2} (\sin\frac{\pi}{4} + \sin\frac{\pi}{4} + \sin\frac{\pi}{4}) = \frac{3}{2} \sin\frac{\pi}{4}$$

$$\cos A = \frac{1}{\sqrt{2}} \xrightarrow{\text{با توجه به A}} A = \frac{\pi}{4}$$

$$\sin B = \frac{1}{\sqrt{2}} \xrightarrow{\text{با توجه به B}} B = \frac{\pi}{4}$$

$$\frac{3\pi}{4} \leftarrow \text{میزبان: C}$$

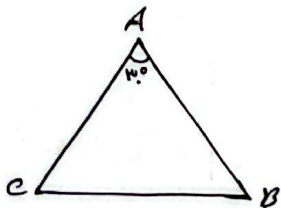
$$\Rightarrow S_{ABC} = \frac{1}{2} (\frac{w+\sqrt{2}}{\sqrt{2}}) = \frac{w+\sqrt{2}}{2}$$

$$\overline{AB} = \sqrt{(\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}})^2 + (\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}})^2} = \sqrt{2}$$

$$\overline{BC} = \sqrt{(-\frac{\sqrt{2}}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}} + 1)^2} = \sqrt{2}$$

$$\overline{AC} = \sqrt{(\frac{1}{\sqrt{2}})^2 + (\frac{\sqrt{2}}{\sqrt{2}} + 1)^2} = \sqrt{2+\sqrt{2}}$$

$$\Rightarrow P_{ABC} = \sqrt{2} + \sqrt{2} + \sqrt{2+\sqrt{2}}$$



$$S = \frac{1}{2} \times AC \times AB \times \sin A$$

$$\hookrightarrow \frac{1}{2} \times \log_2^w \times \log_2^{14} = \frac{1}{2} \log_2^w \times \log_2^2$$

$$S = \frac{1}{2} \times 3 \times 6 \times \log_2^w \times \log_2^2 = 3$$

$$x - \cos\alpha = 0 \Rightarrow x = \cos\alpha \Rightarrow f(\cos\alpha) = 1$$

$$\Rightarrow \cos^2\alpha - \cos^2\alpha - \sin^2\alpha (\cos^2\alpha - 1) = 1 \Rightarrow \cos^2\alpha (\cos^2\alpha + \sin^2\alpha) = 1$$

$$\cos^2\alpha (\cos^2\alpha - 1) = 1$$

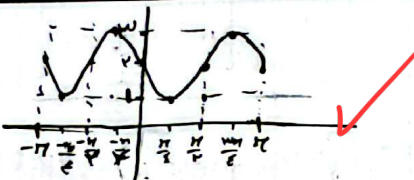
$$\Rightarrow \cos^2\alpha = 1$$

$$\sin^2\alpha = 1 - \cos^2\alpha = 0 \Rightarrow \sin\alpha = 0$$

الف) $y = -2 \sin \alpha \cos \alpha + 2 \quad [-\pi, \pi]$

$$= -\sin 2\alpha + 2$$

$T = \pi$ ✓



✓

ب) $y = 2 \cos(\pi x + \frac{\pi}{2}) + 1$

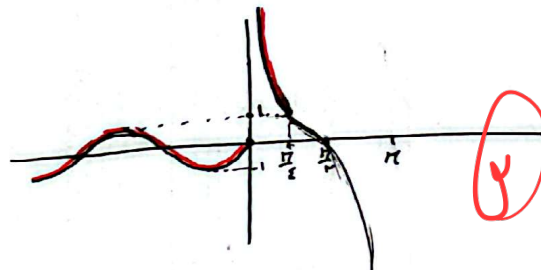
$\Rightarrow y = -2 \sin \pi x + 1 \rightarrow T = 1$ ✓



6

$$f(x) = \begin{cases} ax & ; -\infty < x < \frac{\pi}{2} \\ \sin x & ; x \leq 0 \end{cases}$$

$\Rightarrow R_f = [-1, +\infty)$ ✓



✓

7

$f(x) = b \sin(cx) \cos(cx) + a \Rightarrow f(x) = \frac{b}{2} \sin(2cx) + a$

$$\frac{1}{2} \sin(2cx)$$

$f(x) = \frac{b}{2} \sin(2cx) + a \Rightarrow \begin{cases} \frac{b}{2} + a = 1 \\ -\frac{b}{2} + a = 0 \end{cases}$
 $2a = 1 \Rightarrow a = \frac{1}{2}, |b| = 1$

✓

$T = \frac{\pi}{c} - \frac{\pi}{c} = \frac{2\pi}{c} = \frac{2\pi}{|c|} \Rightarrow |c| = \frac{\omega}{2} \xrightarrow{c>0} \frac{\omega}{2} = c$

$b = 1 : (\frac{\pi}{2}, 1)$ نقطة / من 0 إلى 1

$\frac{bc}{a} = \frac{1 \times \frac{\omega}{2}}{\frac{1}{2}} = \boxed{\omega}$ ✓

8

$T = \pi = \frac{2\pi}{|b|} \Rightarrow |b| = 2 \Rightarrow b = 2$

$f(x) = -2 \cos 2x - 1$

$-2 \cos 2x - 1 = 0 \Rightarrow \cos 2x = -\frac{1}{2}$

✓

$\begin{cases} |a| + c = 2 \\ -|a| + c = -1 \end{cases}$

$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x} \Rightarrow -\frac{1}{2} = \frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - \tan^2 x$

9

$c = -1, |a| = 3 \rightarrow \{2, -4\}$

$\Rightarrow a = -3$
 $\boxed{\frac{2\sqrt{2}}{3}}$ ✓

$\frac{2}{3} \tan^2 x = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$
 $\tan^2 x = \pm \sqrt{\frac{1}{3}}$

$T = \pi \Rightarrow f(x) = f(x + \pi) \rightarrow -2 \cos 2x + \pi n \in [-1, 1] \Rightarrow -1 \leq -2 \cos 2x + \pi n < 1$
 $n \in \mathbb{Z}$

$\Rightarrow \frac{\pi n}{2} - 1 \leq \pi n < \frac{\pi n}{2} + 1 \Rightarrow \frac{\pi n}{2} \leq n < \frac{\pi n}{2} + 1 \Rightarrow n = 1 \rightarrow x' = -2 \cos 2x + \pi(1) = 1, \omega$

10

$\Rightarrow f(1/2) = f(-1/2)$



$y = -\frac{1}{2}x + 1 \Rightarrow f(1/2) = -\frac{1}{2} \times \frac{\pi}{2} + 1 = \boxed{\frac{1}{2}}$ ✓

✓