(۱

$\frac{OA}{OB} = \frac{\sin\alpha}{OT} \xrightarrow{OT=OH=1} \sin\alpha = \frac{1}{OB}$

$AB = OB - OA = \frac{1}{\sin\alpha} - 1$ ✓ (۲)

(۲

$S_{AB\hat{O}} = 2S_{ABC} \longrightarrow \sin 60 \times AB \times AD = 2\sin 30 \times AD \times AC \longrightarrow \frac{AB}{AC} = \frac{2}{\sqrt{2}} = \sqrt{2}$ ✓ (۲)

(۳

$A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), B\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right), C(0,-1)$

$S = \frac{1}{2}\left|\frac{1}{2}\left(\frac{1}{2}+1\right) + \frac{-\sqrt{3}}{2}\left(-1-\frac{\sqrt{3}}{2}\right) + 0\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\right| = \frac{3}{4} + \frac{\sqrt{3}}{4}$ ✓

$AB = \sqrt{\frac{(1+\sqrt{3})^2}{4} + \frac{(\sqrt{3}-1)^2}{4}} = \sqrt{2}$ ، $AC = \sqrt{\frac{1}{4} + \frac{(\sqrt{3}+2)^2}{4}} = \sqrt{2+\sqrt{3}}$ ،

$BC = \sqrt{\frac{3}{4} + \frac{9}{36}} = \sqrt{3}$

$P = \sqrt{2} + \sqrt{3} + \sqrt{2+\sqrt{3}}$ ✓ (۲)

(۴

$a^2 = b^2 + c^2 - 2bc\cos(30) \longrightarrow c^2 - \sqrt{3}bc + b^2 - a^2 = 0 \longrightarrow c = \frac{\sqrt{3}b \pm \sqrt{4a^2 - b^2}}{2}$ (۲)

$S = \frac{1}{2}\sin A \times bc = \frac{b^2\sqrt{3} \pm b\sqrt{4a^2-b^2}}{4} \longrightarrow S = 2\sqrt{3}\left(\log_3^2\right)^2 + \sqrt{9 - 4\left(\log_3^2\right)^4}$

$S = \frac{1}{2} \times AB \times AC \times \sin 30 = \frac{1}{2} \times \log_2^{16} \times \log_3^{81} = \frac{1}{2} \times 4 \times 4 \times \log_2^3 \times \log_3^2 = 8$

(۵

$2\cos^4\alpha - \cos^2\alpha - \sin^2\alpha(2\sin^2\alpha - 1) = 1 \longrightarrow 2\cos^4\alpha - \cos^2\alpha - (1-\cos\alpha)(2(1-\cos^2\alpha)-1) = 1 \longrightarrow$

$2\cos^4\alpha - \cos^2\alpha - (2\cos^4\alpha - 2\cos^2\alpha + 1) = 1 \longrightarrow 2\cos^2\alpha - 1 = 1 \longrightarrow \cos^2\alpha = 1 \longrightarrow \cos\alpha = \pm 1$ (۲)

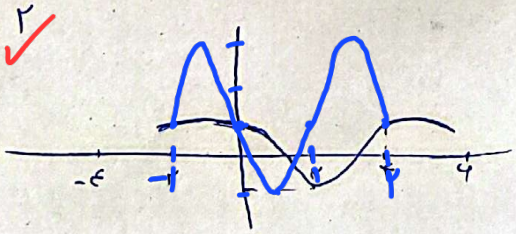
$\sin 2\alpha = 2\cos\alpha\sin\alpha \xrightarrow{\sin\alpha = 0} \sin 2\alpha = 0$ ✓

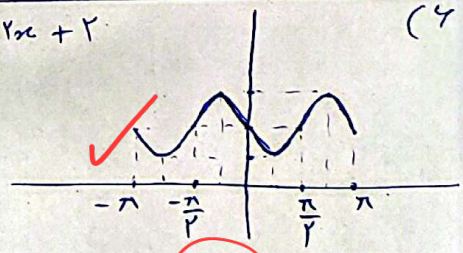
(۶

الف) $T = \frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$ ✓ $y = -2\sin x\cos x + 2 \longrightarrow y = -\sin 2x + 2$

ب) $y = 2\cos\pi\left(x + \frac{1}{2}\right) + 1 = 2\cos\left(\pi x + \frac{\pi}{2}\right) + 1 = -2\sin\pi x + 1$

$T = \frac{2\pi}{|b|} = \frac{2\pi}{\pi} = 2$ ✓

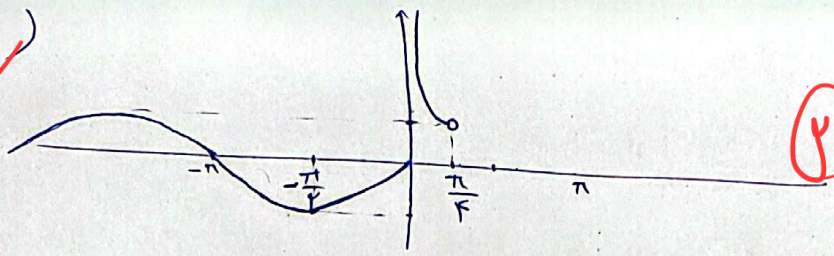




(۱۵)

(۷

$R_f = [-1, +\infty)$ ✓



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(۸

$a + b\sin(cx)\cos(cx)\cos(2cx) \longrightarrow a + b\frac{\sin^2(cx)}{2}\cos(2cx) \longrightarrow$

$a + \frac{b}{4}\sin(4cx)$

باید $b \neq 0$ باشد تا برد از a تایی باشد و $a \neq 2$ باشد تا نمودار ۲ واحد بیاید بالا

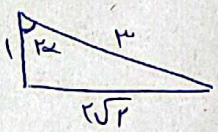
$\frac{2\pi}{4c} = \frac{\pi}{10} \longrightarrow c = \frac{5}{4}$ $\qquad \frac{bc}{a} = \frac{8 \times \frac{5}{4}}{2} = 5$ ✓

۲

(۹

$\frac{2\pi}{|b|} = \frac{2\pi}{\pi} = 2 \longrightarrow f(x) = -3\cos 2x - 1 \longrightarrow f(\alpha) = -3\cos 2\alpha - 1 = 0 \longrightarrow$

$\cos 2\alpha = -\frac{1}{3}$



$\longrightarrow |\tan 2\alpha| = \frac{2\sqrt{2}}{1} = \sqrt{8}$ ✓

۲

(۱۰

$f(x) = f(x+T) \longrightarrow T = 4$

$f(-44/5) = f(1/5)$ $\qquad 0 < x < 2 : \frac{x}{2} + 1$ $\Big\} \longrightarrow f(1/5) = \frac{11}{4} = f(-44/5)$ ✓

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