پرسمان شمشیری - تکلیف شماره (۱۴)

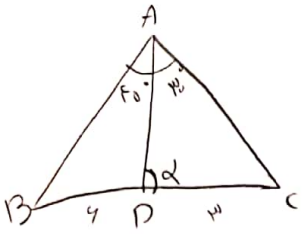
سوال ۱)

$$\tan\left(\frac{\pi}{2}-\alpha\right)=\cot\alpha=\frac{BT}{OT}\xrightarrow{OT=1} BT=\cot\alpha=\frac{\cos\alpha}{\sin\alpha}$$

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\frac{BT}{OB}\rightarrow OB=\frac{BT}{\cos\alpha}=\frac{\frac{\cos\alpha}{\sin\alpha}}{\cos\alpha}=\boxed{\frac{1}{\sin\alpha}}$$

$$\rightarrow AB=OB-OA=\boxed{\frac{1}{\sin\alpha}-1}$$

سوال ۲)

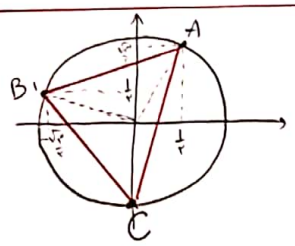


$$\frac{BD}{\sin 45}=\frac{AB}{\sin(\pi-\alpha)}\rightarrow\frac{6}{\frac{\sqrt{2}}{2}}=\frac{AB}{\sin\alpha}\rightarrow AB=\frac{12}{\sqrt{2}}\sin\alpha$$

$$\frac{DC}{\sin 30}=\frac{AC}{\sin\alpha}\rightarrow\frac{3}{\frac{1}{2}}=\frac{AC}{\sin\alpha}\rightarrow AC=6\sin\alpha$$

$$\rightarrow\frac{AB}{AC}=\frac{\frac{12}{\sqrt{2}}\sin\alpha}{6\sin\alpha}=\frac{2}{\sqrt{2}}=\boxed{\sqrt{2}}$$

سوال ۳)



$$\rightarrow A\left|\begin{matrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{matrix}\right.\quad B\left|\begin{matrix}-\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{matrix}\right.\quad C\left|\begin{matrix}0\\ -1\end{matrix}\right.$$

الف)

$$S_{ABC}=\left|\begin{matrix}\frac{1}{2} & \frac{\sqrt{3}}{2} & 1\\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 1\\ 0 & -1 & 1\end{matrix}\right|\rightarrow S=\frac{\left(\frac{1}{4}+0+\frac{\sqrt{3}}{2}\right)-\left(0-\frac{1}{2}+0\right)}{2}=\frac{\frac{1+2\sqrt{3}+2}{4}}{2}$$

$$=\boxed{\frac{3+2\sqrt{3}}{8}}$$

ب)

$$AC=\sqrt{\left(\frac{1}{2}-0\right)^2+\left(\frac{\sqrt{3}}{2}+1\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}+1+\sqrt{3}}=\boxed{\sqrt{2+\sqrt{3}}}$$

$$AB=\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}+\frac{\sqrt{3}}{2}+\frac{3}{4}+\frac{1}{4}-\frac{\sqrt{3}}{2}}=\boxed{\sqrt{2}}$$

$$BC=\sqrt{\left(-\frac{\sqrt{3}}{2}-0\right)^2+\left(\frac{1}{2}+1\right)^2}=\sqrt{\frac{3}{4}+\frac{1}{4}+1+1}=\boxed{\sqrt{3}}$$

$$\text{محیط}=\sqrt{2}+\sqrt{3}+\sqrt{2+\sqrt{3}}$$

سوال ۴)

سوال ۵)

$$2\cos^4\alpha-\cos^2\alpha-\sin^2\alpha(2\sin^2\alpha-1)=\cos^2\alpha(2\cos^2\alpha-1)-\sin^2\alpha(2\sin^2\alpha-1)$$

$$=\cos^2\alpha(\cos 2\alpha)-\sin^2\alpha(-\cos 2\alpha)\rightarrow\cos 2\alpha(\cos^2\alpha+\sin^2\alpha)=\cos 2\alpha=1$$

$$\rightarrow\boxed{\sin 2\alpha=0}$$