۱۸٫۷۵

مثلثات ـ پاسخ تشریحی تکلیف

۱ـ $\sin\alpha = \cos 1$ $\quad OB^2 = OT^2 + BT^2$

$\tan 1 - 1 = AB$ $\quad AB = \frac{1}{\sin\alpha} - 1$ (۱)

$OB^2 = 1 + \cot^2\alpha$ $\quad \frac{\sin 1}{\cos 1} \to \frac{\sqrt{1-\sin a}}{\sin a} - 1 = AB$

$1 + \cot^2\alpha = \frac{1}{\sin^2\alpha}$ $\quad \sin 1 + \cos 1 = 1^2 \to \sin 1 = \sqrt{1-\cos 1}$

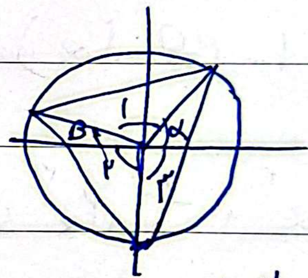
$\to OB = \frac{1}{\sin\alpha}$

۲ـ $\frac{12}{\sqrt{2}} = \frac{6}{\frac{\sqrt{2}}{2}} = \frac{AB}{\underbrace{\sin(180-\alpha)}_{\sin\alpha}} \to AB = \frac{12\sin\alpha}{\sqrt{2}}$ ـ۲

$\frac{AB}{AC} = \frac{\frac{12\sin a}{\sqrt{2}}}{\frac{6\sin\alpha}{1}} = \frac{12}{6\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ ✓ (۲)

۳ـ

الف)



$\alpha + \beta = 90$

$S_1 = \frac{1}{2}\sin 90 \times OA \times OB = \frac{1}{2}$

$S_2 = \frac{1}{2}\sin 120 \times OB \times OC = \frac{\sqrt{3}}{4}$

$S_3 = \frac{1}{2}\sin 150 \times OC \times OA = \frac{1}{4}$

$S_{کل} = \frac{1}{4} + \frac{1}{2} + \frac{\sqrt{3}}{4} = \frac{3+\sqrt{3}}{4}$ ✓ (۲)

$A(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$B(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

$C = (0, -1)$

ب)

$AB = \sqrt{(\frac{1+\sqrt{3}}{2})^2 + (\frac{\sqrt{3}-1}{2})^2} = \sqrt{\frac{8}{4}} = \sqrt{2}$

$AC = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}+2}{2})^2} = \sqrt{\frac{8+4\sqrt{3}}{4}} = \sqrt{2+\sqrt{3}}$

$BC = \sqrt{\frac{3}{4} + (\frac{1}{2}+1)^2} = \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{3}$

محیط $= \sqrt{2} + \sqrt{3} + \sqrt{2+\sqrt{3}}$ ✓

PAYCO

DATE :

$AC = \log_{\sqrt{3}}^{12} = \log_{3}^{6} = 6\log_{3}^{2}$ -٤

$BC = \log_{2}^{27} = \log_{2}^{3^3} = 3\log_{2}^{3} = \frac{3}{\log_{3}^{2}}$ (٢)

$S_{ABC} = \sin\hat{A} \times \frac{1}{2} \times AB \times AC = 6\log_{3}^{2} \times \frac{3}{\log_{3}^{2}} \times \frac{1}{2} \times \sin 30 =$

$S_{A\hat{B}C} = \frac{1}{2} \times \frac{1}{2} \times 6 \times 3 < 3$ ✓

$x - \cos\alpha = 0 \longrightarrow x = \cos\alpha$ -ب

$2\cos^4\alpha - \cos^2\alpha - \sin^2\alpha(2\sin^2\alpha - 1) = \cos^2\alpha(2\cos^2\alpha - 1) +$

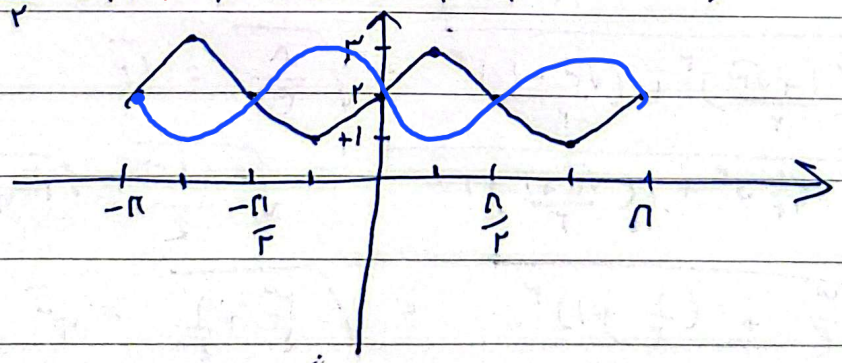
$\sin^2\alpha(1 - 2\sin^2\alpha) = \cos 2\alpha(\cos^2\alpha + \sin^2\alpha) = 1 \rightarrow \cos 2\alpha < 1$

(٢)

$\cos 2\alpha < 1 + \sin 2\alpha < 1 \longrightarrow \sin 2\alpha = 0$ ✓

ج) $-2\sin\alpha\cos\alpha + 2 = -\sin 2\alpha + 2$ $[-\pi, \pi]$ -ج

$T = \frac{2\pi}{2} = \pi$ ✓ , $max = |-1| + 2 < 3$, $Min = -|-1| + 2 = 1$



١٧٥

PAYCO

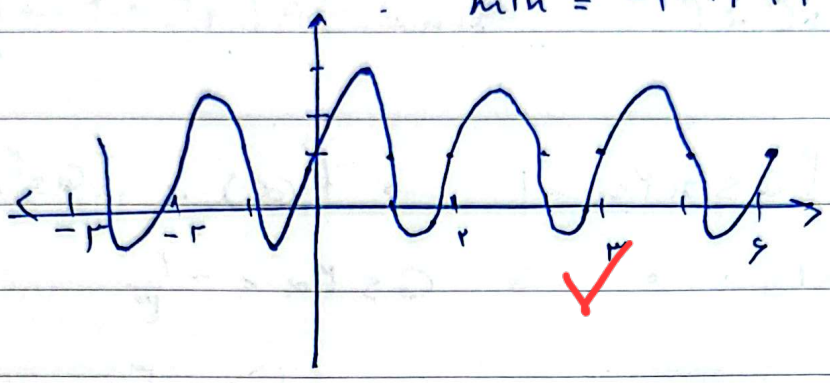
DATE :

$\rightarrow) \; 2\cos\left(x+\frac{1}{2}\right)\pi + 1 = 2\cos\left(\frac{\pi}{2}+\pi x\right)+1$

$= -2\sin \pi x + 1$

$T = \frac{2\pi}{\pi} = 2$ ✓ $\longrightarrow$ max $= |-2|+1 = 3$

min $= -|-2|+1 = -1$



✓

$f(x) = a + b\sin(\pi x)\cdot\cos(\pi x)\cdot\cos(2\pi x) = -1$

$a + \frac{b}{2}\sin(2\pi x)\cdot\cos(2\pi x) = a + \frac{b}{4}\sin(4\pi x)$

$T = \frac{2\pi}{10} = \frac{\pi}{5} = \frac{4\pi}{10} = \frac{2\pi}{5} = \frac{2\pi}{4c} \longrightarrow c = \frac{5}{4}$

min $= a - \left|\frac{b}{4}\right| = 0 \longrightarrow a = \left|\frac{b}{4}\right| \longrightarrow ab > 0$

max $= a + \left|\frac{b}{4}\right| = 4 \longrightarrow 2a < 4 \longrightarrow a = 2$

$\frac{b}{4} = 2 \rightarrow b = 8$

$\frac{bc}{a} = \frac{8 \times \frac{5}{4}}{2} = 5$ ✓

✓

PAYCO

DATE :

$f(x) = a\cos bx + c$ -9

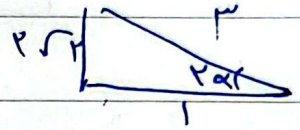
$T = \frac{2\pi}{b} = \pi \rightarrow b = 2$

$max = |a| + c = 2$ $\quad min = -|a| + c = -4 \rightarrow c = -1$
$a = -3$

$f(x) = -3\cos x 2x - 1 \rightarrow f(\alpha) = -3\cos^2\alpha - 1 < 0$

$-3\cos 2\alpha - 1 < 0 \rightarrow \cos 2\alpha < -\frac{1}{3}$

$|\tan 2\alpha| = 2\sqrt{2}$ ✓ (۲)



$y = f(x) = f(x \pm T)$ -10

$f(-2\pi, 0) = f(x - 3\pi) \rightarrow x - 3\pi = -2\pi, 0$
$f(1,0)$

$f(1,0) = -\frac{1}{2}x + 1$

$f(1,0) = -\frac{1}{2}\left(\frac{\pi}{2}\right) + 1 = -\frac{\pi}{4} + 1 = \frac{1}{4} = 0.25$ ✓

(۲)