فرناز زنجیل

۱۲٫۵

۱-

$x=۰ \to \tan(۰+\frac{\pi}{۴})=۱ \Rightarrow B=(۰,۱)$

$y=\tan(-۲x+\frac{\pi}{۴})$ $\quad T=\frac{\pi}{|k|}=\frac{\pi}{|-۲|}=\frac{\pi}{۲} \Rightarrow AC=\frac{\pi}{۲}$ $\quad S_{ABC}=۱\times\frac{\pi}{۲}\times\frac{۱}{۲}=\frac{\pi}{۴}$

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۲-

$x^۲-\sin(\alpha)x-\frac{۱}{۴}(۱-\sin^۲\alpha)=۰$

$\Delta=b^۲-۴ac=\sin^۲\alpha-۴\times(-\frac{۱}{۴}\times(۱-\sin^۲\alpha))=\sin^۲\alpha+۱-\sin^۲\alpha=۱$

$\sin^۲\alpha-۴\times(-\frac{۱}{۴}\times(\cos^۲\alpha))=\sin^۲\alpha+\cos^۲\alpha=۱$

$\frac{-b\pm\sqrt{\Delta}}{۲a}=\frac{\sin\alpha\pm۱}{۲}=\frac{۳}{۴}$ $\quad \sin\alpha=\frac{۳}{۲}\mp۱ \Rightarrow \sin\alpha=\frac{۱}{۲}$

$۱+\tan^۲\alpha=\frac{۱}{\cos^۲\alpha} \Rightarrow ۱+\tan^۲\alpha=\frac{۹}{۸}$

$\cos^۲\alpha=۱-\sin^۲\alpha=۱-\frac{۱}{۹}=\frac{۸}{۹}$

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۳-

$\log\frac{\sin\alpha}{-\cos\alpha}=\log ۳$ $\quad \sin\alpha=-۳\cos\alpha \Rightarrow \sin^۲\alpha=۹\cos\alpha \Rightarrow ۹\cos\alpha=۱-\cos^۲\alpha \Rightarrow \cos^۲\alpha+۹\cos\alpha-۱=۰$

$\sin^۲\alpha=۱-\cos^۲\alpha$

$\Rightarrow \Delta=۸۱-۴\times(-۱)=۸۵ \Rightarrow \frac{-b\pm\sqrt{۸۵}}{۲}=\frac{-۹\pm\sqrt{۸۵}}{۲} \Rightarrow \cos\alpha=\frac{-۹+\sqrt{۸۵}}{۲}$ $\quad \sin\alpha=-۳\times(\frac{-۹+\sqrt{۸۵}}{۲})$

$-۳\cos\alpha-\cos\alpha=-۴\cos\alpha=-۴\times(\frac{-۹+\sqrt{۸۵}}{۲})=۱۸-۲\sqrt{۸۵}$

$\log \sin\alpha \to \sin\alpha>۰$

$\log -\cos\alpha \to -\cos\alpha>۰ \to \cos\alpha<۰$ $\Big\}$ ناحیه دوم α

$\log\frac{-\sin\alpha}{\cos\alpha}=\log ۳ \to \tan\alpha=-۳$

$\sin\alpha=\frac{۱}{\sqrt{۱۰}}$

$\cos\alpha=-\frac{۳}{\sqrt{۱۰}}$

$\sin\alpha-\cos\alpha=\frac{۱}{\sqrt{۱۰}}-(-\frac{۳}{\sqrt{۱۰}})=\frac{۴}{\sqrt{۱۰}}$

$\sin^4\theta + \cos^2\theta = \frac{4}{5} \quad (\sin^2\theta)^2 + \cos^2\theta = \frac{4}{5}$

$(1 - \cos^2\theta)^2 + \cos^2\theta = \frac{4}{5} \rightarrow \cos^4\theta - \cos^2\theta = -\frac{1}{5}$

$\cos^4\theta - (1 - \sin^2\theta) = -\frac{1}{5} \rightarrow \cos^4\theta + \sin^2\theta = \frac{4}{5}$

$\sin^4\theta + \cos^2\theta = 4/5 \quad \sin^4\theta = 1 - \cos^4\theta \Rightarrow \cos^4\theta - \cos^2\theta - 1/5 = 0$ ۴-

$\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm \sqrt{1 + 4/5}}{2} = \frac{1 \pm 3\sqrt{5}/5}{2} = \frac{5 \pm 3\sqrt{5}}{10} \Rightarrow \cos^2\theta = \frac{5 - 3\sqrt{5}}{10}$

$\Rightarrow \cos^4\theta = \frac{5 - 3\sqrt{5}}{10} + \frac{2}{10} = \frac{7 - 3\sqrt{5}}{10} \quad \sin^2\theta = 1 - \left(\frac{5 - 3\sqrt{5}}{10}\right) = \frac{5 + 3\sqrt{5}}{10} \Rightarrow \cos^4\theta + \sin^2\theta = \frac{12}{10} = 1/2$

$\frac{\sin x(3 + \cos x)}{2(1 - \cos^2 x)} = \frac{3 + \cos x}{2\sin x} < 0$ $\quad 3 \pm 1 \Rightarrow$ همواره مثبت ۵-

$\sin^2 x$

$\Rightarrow \sin x < 0$

در ربع سوم است $\Rightarrow$

$\sin x \times \frac{\sin x}{\cos x} - \frac{1}{\cos x} > 0 \Rightarrow \frac{-\cos^2 x}{\cos x} = -\cos x > 0 \Rightarrow \cos x < 0$

۳

$\frac{2}{\sin\alpha} = -\frac{3}{\cos\alpha} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \cot\alpha = -\frac{3}{2} \Rightarrow \tan\alpha + \cot\alpha = -\frac{3}{2} - \frac{2}{3} = \frac{-9-4}{6} = -\frac{13}{6}$ -۶

۲

$\sin 50^\circ = \cos 40^\circ = \cos 2\times 20^\circ = \cos^2 20^\circ - \sin^2 20^\circ \Rightarrow \sqrt{1+\cos^2 20^\circ - \sin^2 20^\circ} = \sqrt{2}\cos 20^\circ$ -۷

$\hookrightarrow 2\cos^2 20^\circ$

$\sin A + \sin B = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \Rightarrow 2\sin 30\cos 20 = \cos 20^\circ \Rightarrow \frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ} = \sqrt{2}$

$\frac{1}{2}$

۲

$1-2\sin^2 5^\circ = \sin^2 5^\circ + \cos^2 5^\circ - 2\sin^2 5^\circ = \cos^2 5^\circ - \sin^2 5^\circ = \cos 10^\circ \rightarrow \sqrt{1-\sin^2 10}$ -۸

$\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ} = \cos 40^\circ = \cos^2 20^\circ - \sin^2 20^\circ$ $\hookrightarrow 2\sin 10\cos^2 10^\circ = \cos^4 10^\circ + \sin^4 10^\circ$ $\hookrightarrow (1-\sin^2 10)^2 = 2\sin^4 10 + 1 - 2\sin^2 10^\circ$

$\cancel{(\cos^2 10-\sin^2 10)^2 \cos^4 10 + \sin^4 10 + 2\sin^2 10^\circ\cos^2 10^\circ}$

$2\cos^2 10^\circ - 1 = \cos^2 10^\circ + \cos^2 10 - 1$ $\Rightarrow (2\cos^2 10^\circ - \cos 10^\circ)(\cos^4 10^\circ + \sin^4 10^\circ) =$

$\hookrightarrow 2 - \sin^2 10^\circ = 1 - \sin^2 10^\circ - \sin^2 10^\circ$

$1 - 2\sin^2 5 = \cos 10$

$2\cos^2 10 - 1 = \cos 20$

$\frac{1-\tan^2 20}{1+\tan^2 20} = \cos 40$

$\Rightarrow (1-\sin^2 10)(\sqrt{1-\sin^2 10^\circ})(2\sin^4 10 + 1 - 2\sin^2 10)$

$\cos 10 \times \cos 20 \times \cos 40 \times \frac{\sin 10}{\sin 10} = \frac{1}{8}\cot 10$

۰/۵

-۹

$\sin B\sin C = \cos^2\frac{\hat{A}}{2}$ $\quad \sin C = \cos 18 \rightarrow \cos^2 18^\circ = \cos^2\frac{\hat{A}}{2} \Rightarrow A = 36^\circ$ -۱۰

$\hookrightarrow \cos 18^\circ$ $\quad \rightarrow 18 + 72 + 90 = 180^\circ$

$180 - 72 = 108$

$\frac{108}{2} = 54$

۲