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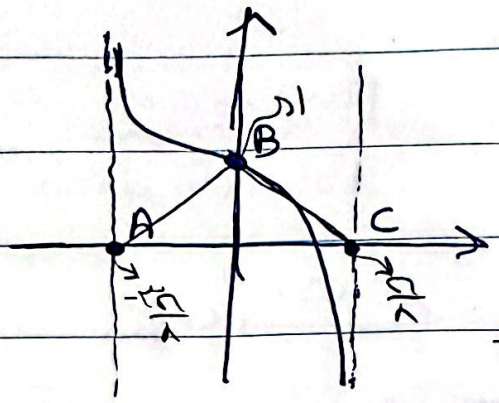
مدیم توکلی دوازدهم A صبح تکلیف ۵۱

سوال ۱) $y = \tan(-2x + \frac{\pi}{4})$ ABC تشکیل می‌دهند؟



$B \Rightarrow x=0 \rightarrow \tan(\frac{\pi}{4}) = 1 \rightarrow B = (0,1)$

$A, C \Rightarrow \tan(-2x+\frac{\pi}{4}) = 0$

$\rightarrow \tan(-2x+\frac{\pi}{4}) = \tan(\pi) \Rightarrow -2x = \frac{3\pi}{4}$

$\Rightarrow x = -\frac{3\pi}{8} \rightarrow A = (-\frac{3\pi}{8}, 0)$

$\rightarrow \tan(-2x+\frac{\pi}{4}) = \tan(0) \rightarrow -2x = -\frac{\pi}{4} \rightarrow x = +\frac{\pi}{8}$

$\Rightarrow C = (+\frac{\pi}{8}, 0)$

$S_{AB\hat{C}} = \frac{1 \times \frac{4\pi}{8}}{2} = \boxed{\frac{\pi}{4}}$

سوال ۲) $x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0$

$x = \frac{b}{a} \rightarrow \frac{4}{9} - \frac{2}{3}(\sin\alpha) - \frac{1}{4}(1-\sin^2\alpha) = 0$

$\times 36 \rightarrow 16 - 24\sin\alpha - 9(1-\sin^2\alpha) = 0 \Rightarrow 9\sin^2\alpha - 24\sin\alpha + 7 = 0$

$\Rightarrow \sin\alpha = \frac{24 \pm 18}{18} \nearrow \sin\alpha = \frac{42}{18}$ قق

$\searrow \sin\alpha = \frac{1}{3}$ قق $\quad \sin^2\alpha + \cos^2 = 1 \Rightarrow \cos^2 = \frac{8}{9}$

$\Rightarrow \cos\alpha = \frac{2\sqrt{2}}{3}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \longrightarrow 1 + \tan^2\alpha = \frac{1}{\frac{8}{9}} \rightarrow \boxed{\frac{9}{8}}$

سوال ۳) $\log(+\sin\alpha) - \log(-\cos\alpha) = \log 3$ $\quad \sin\alpha - \cos\alpha = ?$

$\Rightarrow \log -\frac{\sin\alpha}{\cos\alpha} = \log 3 \longrightarrow -\tan\alpha = 3 \rightarrow \tan\alpha = -3$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \longrightarrow 1+9 = \frac{1}{\cos^2\alpha} \Rightarrow \cos^2\alpha = \frac{1}{10} \rightarrow \cos\alpha = \pm\frac{\sqrt{10}}{10}$

$\cos\alpha = -\frac{\sqrt{10}}{10}$ ← $\cos\alpha$ منفی است ← چون در ربع چهارم $-\cos\alpha$ باید مثبت باشد پس

و $\sin\alpha$ مثبت است

$\sin^2\alpha + \cos^2\alpha = 1 \longrightarrow \sin^2\alpha = 1 - \frac{1}{10} \Rightarrow \sin^2\alpha = \frac{9}{10} \rightarrow \sin\alpha = +\frac{3\sqrt{10}}{10}$

$\sin\alpha - \cos\alpha = \frac{3\sqrt{10}}{10} - \left(-\frac{\sqrt{10}}{10}\right) = \frac{4\sqrt{10}}{10} = \boxed{\frac{2\sqrt{10}}{5}}$

سوال ۴) $\sin^4\theta + \cos^2\theta = \frac{4}{5}$ $\quad \cos^4\theta + \sin^2\theta = ?$

$\hookrightarrow \sin^4\theta + 1 - \sin^2\theta = \frac{4}{5} \Rightarrow \sin^2\theta(\underbrace{\sin^2\theta - 1}_{-\cos^2\theta}) = -\frac{1}{5}$

$\Rightarrow \sin^2\theta\cos^2\theta = \frac{1}{5}$

$\cos^4\theta + \sin^2\theta \longrightarrow \cos^4\theta + 1 - \cos^2\theta \Rightarrow \underbrace{\cos^2\theta(\underbrace{\cos^2\theta - 1}_{-\sin^2\theta}) + 1}_{-\sin^2\theta\cos^2\theta} = \boxed{\frac{4}{5}}$

$\underbrace{\quad}_{-\frac{1}{5}}$

سوال ۵) $\frac{3\sin\alpha + \sin\alpha\cos\alpha}{2 - 2\cos^2\alpha} < 0 \qquad \sin\alpha\tan\alpha - \frac{1}{\cos\alpha} > 0$ کدام ناصحیح؟

$2(1-\cos^2\alpha) = 2\sin^2\alpha$

$\hookrightarrow \frac{\sin\alpha(3+\cos\alpha)}{2\sin^2\alpha} < 0 \rightarrow \frac{\text{همواره مثبت}}{\text{همواره مثبت}} < 0 \Rightarrow \sin\alpha < 0$

* ادامه حل در سند بعد!

$\sin x \tan x - \frac{1}{\cos x} > 0 \rightarrow \frac{\sin^2 x - 1}{\cos x} > 0 \Rightarrow \frac{-\cos^2 x}{\cos x} > 0$

$\Rightarrow -\cos x > 0 \Rightarrow \cos x < 0$

$\sin x$ منفی و $\cos x$ منفی هستند ← ناحیه سوم

سوال ۶)

$\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \quad \tan x + \cot x = ?$

$\rightarrow \frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \rightarrow 2\cos x = -3\sin x \Rightarrow \cos x = -\frac{3}{2}\sin x$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + \frac{9}{4}\sin^2 x = 1 \Rightarrow \frac{13}{4}\sin^2 x = 1$

$\Rightarrow \sin x = \pm\frac{2\sqrt{13}}{13}$ و $\cos x = \pm\frac{3\sqrt{13}}{13}$ ← $\sin x$ و $\cos x$ مختلف العلامت هستند یکی را

منفی در نظر می‌گیریم (فرقی نمی‌کند کدام باشد چون در جواب نهایی تاثیری نخواهد داشت!)

$\tan x + \cot x \rightarrow \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \rightarrow \frac{1}{\frac{2\sqrt{13}}{13} \times \left(-\frac{3\sqrt{13}}{13}\right)} = \frac{1}{-\frac{6}{13}}$

$\Rightarrow \tan x + \cot x = \boxed{-\frac{13}{6}}$

سوال ۷)

$\frac{\sqrt{1+\sin 50}}{\sin 50 + \sin 10} = ?$

$1+\sin 50 \rightarrow \sin^2 25 + \cos^2 25 + 2\sin 25\cos 25$

$\Rightarrow (\sin 25 + \cos 25)^2 \xrightarrow{\text{جذر}}$ صورت $= \sin 25 + \cos 25$

$\sin\alpha + \cos\alpha = \sqrt{2}\sin(\alpha + \frac{\pi}{4}) \rightarrow \sin 25 + \cos 25 = \sqrt{2}\sin 70 = \sqrt{2}\cos 20$

$\sin 50 + \sin 10 \rightarrow \sin(30+20) + \sin(30-20)$ * ادامه حل در صفحه بعد!

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$\Rightarrow \underbrace{\sin 30 \cos 20}_{\frac{1}{2}\cos 20} + \sin 20 \cos 30 + \underbrace{\sin 30 \cos 20}_{\frac{1}{2}\cos 20} - \sin 20 \cos 30 = \cos 20$

$\rightarrow \frac{\sqrt{2}\cos 20}{\cos 20} = \boxed{\sqrt{2}}$

سوال ۸

$(1-2\sin^2\alpha)(2\cos^2 10 - 1)\left(\frac{1-\tan^2 40}{1+\tan^2 40}\right)$

$\frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \cos 2\alpha \Rightarrow \frac{1-\tan^2 40}{1+\tan^2 40} = \cos 80$

$\rightarrow (\cos^2\alpha - \sin^2\alpha)(\cos^2 10 - \sin^2 10)(\cos 80) = (\cos 10)(\cos 20)(\cos 40)$

$\times \sin 10 \rightarrow \underbrace{(\sin 10 \cos 10)}_{\frac{1}{2}\sin 20}(\cos 20)\underbrace{(\cos 40)}_{\frac{1}{4}\sin 80} \rightarrow \frac{\frac{1}{8}\sin 160}{\sin 10} = \boxed{\frac{1}{8}\cot 10}$

سوال ۹

$\sin\alpha + 2\cos\alpha = 2 \qquad 5\cos 2\alpha - 12\cos\alpha = ?$

$\cos 2\alpha = 2\cos^2\alpha - 1 \rightarrow 10\cos^2\alpha - 5 - 12\cos\alpha = 5$

$\rightarrow \sin\alpha = 2 - 2\cos\alpha \rightarrow \sin^2\alpha + \cos^2\alpha = 1 \rightarrow (2-2\cos\alpha)^2 + \cos^2\alpha = 1$

$\rightarrow 4\cos^2\alpha - 8\cos\alpha + 4 + \cos^2\alpha = 1 \rightarrow 5\cos^2\alpha - 8\cos\alpha + 4 = 1$

$\rightarrow 10\cos^2\alpha - 16\cos\alpha = -6 \rightarrow 10\cos^2\alpha - 16\cos\alpha - 5 = -6 - 5 = \boxed{-11}$

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سوال ۱۰) $\sin\hat{B}\sin\hat{C} = \cos^2\frac{\hat{A}}{2}$ $\quad \hat{B}=72$ $\quad \hat{A}=?$

$\cos^2\frac{\hat{A}}{2} = \frac{1+\cos\hat{A}}{2} \rightarrow \cos\hat{A} = \cos(\pi-(B+C)) = -\cos(B+C)$

$\Rightarrow 1-\cos(B+C) = 2\sin B\sin C \rightarrow 1-(\cos B\cos C - \sin B\sin C) = 2\sin B\sin C$

$\Rightarrow 1-\cos B\cos C+\sin B\sin C = 2\sin B\sin C$

$\rightsquigarrow 1-(\cos B\cos C+\sin B\sin C)=0 \Rightarrow 1-\cos(B-C)=0$

$\Rightarrow \cos(B-C)=1 \Rightarrow B-C=0 \Rightarrow \hat{B}=\hat{C}=72^\circ$

$A = 180-(B+C) \rightarrow \hat{A} = 180-(144) = \boxed{36^\circ}$