Subject: حسابان

Date: / /

۱۹,۲۵

1) $y = \tan(-2x + \frac{\pi}{4})$

$x = 0 \rightarrow y = \tan(\frac{\pi}{4}) = 1 \rightarrow B(0,1)$

$T = \frac{\pi}{|a|} = \frac{\pi}{2} \rightarrow T = \overline{AC} = \frac{\pi}{4}$

$S_{\overset{\triangle}{ABC}} = \frac{AC \times (\text{عرض نقطه } B)}{2}$

$S_{\overset{\triangle}{ABC}} = \frac{\frac{\pi}{4} \times 1}{2} = \frac{\pi}{4}$

۲

2) $\left.\begin{array}{l} x = \frac{2}{3} \\ y = 0 \end{array}\right\} \rightarrow \frac{4}{9} - \frac{2\sin\alpha}{3} - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{\times 36} 16 - 24\sin\alpha - 9\cos^2\alpha = 0$

$\xrightarrow{\times(-1)} 9\cos^2\alpha + 24\sin\alpha - 16 = 0 \xrightarrow{\cos^2\alpha = 1 - \sin^2\alpha} 9 - 9\sin^2\alpha + 24\sin\alpha - 16 = 0$

$9\sin^2\alpha - 24\sin\alpha + 7 = 0 \xrightarrow{\text{روش دوم}} \sin^2\alpha - 24\sin\alpha + 63 = 0$

$(\sin\alpha - 21)(\sin\alpha - 3) = 0$

$\sin\alpha = \frac{21}{9} = \frac{7}{3}$ ✗ (طبق $-1 \le \sin\alpha \le 1$)

$\sin\alpha = \frac{3}{9} = \frac{1}{3}$ ✓

۲

$\sin\alpha = \frac{1}{3} \rightarrow \sin^2\alpha + \cos^2\alpha = 1 \rightarrow (\frac{1}{3})^2 + \cos^2\alpha = 1 \rightarrow \cos^2\alpha = \frac{8}{9}$

حل: $1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{9}{8}$

3) $\log(\sin\alpha) - \log(-\cos\alpha) = \log 3 \rightarrow \log\frac{\sin\alpha}{-\cos\alpha} = \log 3$

$\frac{\sin\alpha}{-\cos\alpha} = 3 \rightarrow -\tan\alpha = 3 \rightarrow \tan\alpha = -3$

۲

تعیین علامت $\sin\alpha$ و $\cos\alpha$: طبق دامنه لگاریتم $\rightarrow -\cos\alpha > 0$

$\cos\alpha < 0$

طبق دامنه لگاریتم $\rightarrow \sin\alpha > 0$

Subject: Date: / /

$\tan\alpha = -3 \xrightarrow{1+\tan^2 = \frac{1}{\cos^2}} 1 + (-3)^2 = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{1}{10}$

$\rightarrow \cos\alpha = -\frac{1}{\sqrt{10}}$

$\xrightarrow{\sin^2\alpha+\cos^2\alpha=1} \frac{1}{10} + \sin^2\alpha = 1 \rightarrow \sin\alpha = \frac{3}{\sqrt{10}}$ 1/5

جواب : $\sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}} = \frac{2\sqrt{10}}{5}$

41)

$\sin^4\alpha + \cos^2\alpha = \frac{4}{5} \xrightarrow{\cos^2\alpha = 1-\sin^2\alpha} \sin^4\alpha - \sin^2\alpha = -\frac{1}{5}$

$\sin^2\alpha(\sin^2\alpha - 1) = -\frac{1}{5} \rightarrow \sin^2\alpha(-\cos^2\alpha) = -\frac{1}{5}$

$\rightarrow \sin^2\alpha\cos^2\alpha = \frac{1}{5} \xrightarrow{\times 4} 4\sin^2\alpha\cos^2\alpha = \frac{4}{5} \rightarrow$

2

$(2\sin\alpha\cos\alpha)^2 = \frac{4}{5} \xrightarrow{2\sin\alpha\cos\alpha = \sin 2\alpha} (\sin 2\alpha)^2 = \frac{4}{5}$

$\cos^4\alpha + \sin^2\alpha = ? \xrightarrow{\sin^2\alpha = 1-\cos^2\alpha} \cos^4\alpha + 1 - \cos^2\alpha =$

$\cos^2\alpha(\cos^2\alpha - 1) + 1 = \cos^2\alpha(-\sin^2\alpha) + 1 \rightarrow$

$\frac{-\cos^2\alpha\sin^2\alpha \times 4}{4} + 1 = \frac{-1}{4}(2\sin\alpha\cos\alpha)^2 + 1 \xrightarrow{2\sin\alpha\cos\alpha = \sin 2\alpha}$

$-\frac{\sin^2 2\alpha}{4} + 1 \xrightarrow{\sin^2 2\alpha = \frac{4}{5}} \frac{-\frac{4}{5}}{4} + 1 = \frac{4}{5}$

s.a.m

Subject: Date: / /

5) $\dfrac{3\sin x + \sin x \cos x}{2 - 2\cos^2\alpha} < 0 \rightarrow \dfrac{\sin\alpha(3 + \cos\alpha)}{2(1 - \cos^2\alpha)} < 0$

(همواره ⊕ (زیرا: $-1 < \cos < 1$))

$\xrightarrow{1 - \cos^2\alpha = \sin^2\alpha} \dfrac{\sin\alpha\,\overbrace{(3 + \cos\alpha)}}{\underbrace{2\sin^2\alpha}_{\text{همواره }+}} < 0 \rightarrow \sin\alpha < 0$

۳ ✓

$\sin\alpha \tan\alpha - \dfrac{1}{\cos\alpha} > 0 \rightarrow \dfrac{\sin^2\alpha}{\cos\alpha} - \dfrac{1}{\cos\alpha} > 0 \rightarrow \dfrac{\sin^2\alpha - 1}{\cos\alpha} >$

همواره ⊖ است

$\xrightarrow{1 - \sin^2\alpha = \cos^2\alpha} \dfrac{-\overbrace{\cos^2\alpha}}{\cos\alpha} > 0 \rightarrow \cos\alpha < 0$

$\left.\begin{array}{l}\sin < 0 \\ \cos\alpha < 0\end{array}\right\} \rightarrow$ ربع ۳

G) $\dfrac{2}{\sin\alpha} + \dfrac{3}{\cos\alpha} = 0 \rightarrow \dfrac{2\cos\alpha + 3\sin\alpha}{\sin\alpha\cos\alpha} = 0 \rightarrow$

$3\sin\alpha + 2\cos\alpha = 0 \rightarrow 3\sin\alpha = -2\cos\alpha \rightarrow$

$\dfrac{\sin\alpha}{\cos\alpha} = -\dfrac{2}{3} \rightarrow \tan\alpha = \dfrac{-2}{3} \rightarrow \cot\alpha = \dfrac{-3}{2}$

$\tan\alpha + \cot\alpha = \dfrac{-2}{3} - \dfrac{3}{2} = \dfrac{-4-9}{6} = \dfrac{-13}{6}$

۳ ✓

s.a.m

71) $A = \dfrac{\sqrt{1+\sin 50}}{\sin 50 + \sin 10} = \dfrac{\sqrt{1+\sin 50}}{\sin(60-10)+\sin 10} = \dfrac{\sqrt{1+\sin 50}}{\frac{\sqrt{3}}{2}\cos 10 - \frac{1}{2}\sin 10 + \sin 10}$

$\dfrac{\sqrt{1+\sin 50}}{\frac{1}{2}\sin 10 + \frac{\sqrt{3}}{2}\cos 10} = \dfrac{\sqrt{1+\sin 50}}{\sin(70)} = \dfrac{\sqrt{1+\sin 50}}{\cos 20}$

$A^2 = \dfrac{1+\sin 50}{\cos^2 20} \xrightarrow{\cos^2 20 = \frac{1+\cos 40}{2}} \dfrac{(1+\sin 50)2}{1+\cos 40} =$

$\sin 50 = \cos 40 \longrightarrow \sqrt{\dfrac{2(1+\cos 40)}{1+\cos 40}} = \sqrt{2} = \sqrt{2}$

1,75

---

81) $(1-2\sin^2 5)(2\cos^2 10 - 1)\left(\dfrac{1-\tan^2 20}{1+\tan^2 20}\right) =$

$\begin{cases} 1-2\sin^2\alpha = \cos 2\alpha \\ 2\cos^2\alpha - 1 = \cos 2\alpha \\ 1+\tan^2\alpha = \dfrac{1}{\cos^2\alpha} \end{cases} \longrightarrow (\cos 10 \times \cos 20)\left(\dfrac{1-\tan^2 20}{\frac{1}{\cos^2 20}}\right)$

2

$\longrightarrow (\cos 10 \cos 20)\left(\dfrac{\frac{\cos^2 20}{\cos^2 20} - \frac{\sin^2 20}{\cos^2 20}}{\frac{1}{\cos^2 20}}\right) = (\cos 10 \times \cos 20)(\cos^2 20 - \sin^2 20$

$\cos 10 \times \cos 20 \times \cos 40 = ? \xrightarrow[\div 8\sin 10]{\times 8\sin 10} \dfrac{2\times 2\times 2\sin 10 \times \cos 10 \times \cos 20 \times \cos 40}{8\sin 10}$

$\dfrac{2\times 2\times \sin 20 \times \cos 20 \times \cos 40}{8\sin 10} = \dfrac{2\times \sin 40 \times \cos 40}{8\sin 10} = \dfrac{\sin 80}{8\sin 10} = \dfrac{\cos 10}{8\sin 10} = \dfrac{\cot}{8}$

Subject: Date: / /

۹) $\sin\alpha + 3\cos\alpha = 2 \rightarrow \sin\alpha = 2 - 3\cos\alpha \rightarrow$

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow (2 - 3\cos\alpha)^2 + \cos^2\alpha = 1 \rightarrow 10\cos^2\alpha - 12\cos\alpha + 3 = 0$

۲ ✓

$5\cos 2\alpha - 12\cos\alpha = 5(2\cos^2\alpha - 1) - 12\cos\alpha \rightarrow$

$\underbrace{10\cos^2\alpha - 12\cos\alpha} - 5 = (-3) - 5 = -8$

۱۰) $\sin\hat{B}\sin\hat{C} = \cos^2\frac{\hat{A}}{2}$ $\hat{B} = 72^\circ$ $\hat{A} = ?$: در مثلث $\overset{\triangle}{ABC}$

$\sin\hat{B}\sin\hat{C} = \frac{1}{2}(\cos\hat{A} + 1)$ ۲ ✓

$\downarrow$

$+\cos(180 - (\hat{B}+\hat{C})) = -\cos(B+C)$

$2\sin\hat{B}\sin\hat{C} = -\cos B\cos C + \sin B\sin C + 1 \longrightarrow$

$\sin\hat{B}\sin\hat{C} + \cos B\cos C = 1 \rightarrow \cos(C - B) = 1 \rightarrow \hat{C} - \hat{B} = 0$

s.a.m $\hat{C} = \hat{B} \rightarrow \hat{B} = 72$ $\hat{A} + \hat{B} + \hat{C} = 180 \rightarrow \hat{A} + 72 + 72 = 180$

$\hat{A} = 36^\circ$