Subject: حسابان

1) $y = \tan(-2x + \frac{\pi}{4})$

$x = 0 \rightarrow y = \tan(\frac{\pi}{4}) = 1 \rightarrow B(0,1)$

$T = \frac{\pi}{|a|} = \frac{\pi}{2} \rightarrow T = \overline{AC} = \frac{\pi}{4}$

$S_{\overset{\triangle}{ABC}} = \frac{AC \times (\text{عرض نقطه } B)}{2}$

$S_{\overset{\triangle}{ABC}} = \frac{\frac{\pi}{4} \times 1}{2} = \frac{\pi}{4}$

---

2) $\left.\begin{matrix} x = \frac{2}{3} \\ y = 0 \end{matrix}\right\} \rightarrow \frac{4}{9} - \frac{2\sin\alpha}{3} - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{\times 36} 16 - 24\sin\alpha - 9\cos^2\alpha = 0$

$\xrightarrow{\times(-1)} 9\cos^2\alpha + 24\sin\alpha - 16 = 0 \xrightarrow{\cos^2\alpha = 1 - \sin^2\alpha} 9 - 9\sin^2\alpha + 24\sin\alpha - 16 = 0$

$9\sin^2\alpha - 24\sin\alpha + 7 = 0 \xrightarrow{\text{روش ضرب}} \sin^2\alpha - 24\sin\alpha + 63 = 0$

$(\sin\alpha - 21)(\sin\alpha - 3) = 0$

$\begin{cases} \sin\alpha = \frac{21}{9} = \frac{7}{3} & \times \; (-1 \leq \sin\alpha \leq 1 \text{ طبق}) \\ \sin\alpha = \frac{3}{9} = \frac{1}{3} & \checkmark \end{cases}$

$\sin\alpha = \frac{1}{3} \rightarrow \sin^2\alpha + \cos^2\alpha = 1 \rightarrow (\frac{1}{3})^2 + \cos^2\alpha = 1$

$\rightarrow \cos^2 = \frac{8}{9}$

جواب: $1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{9}{8}$

$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3 \rightarrow \log\frac{\sin\alpha}{-\cos\alpha} = \log 3$

$\frac{\sin\alpha}{-\cos\alpha} = 3 \rightarrow -\tan\alpha = 3 \rightarrow \tan\alpha = -3$

تعیین علامت $\sin\alpha$, $\cos\alpha$: طبق دامنه لگاریتم $\rightarrow -\cos\alpha > 0$

$\cos\alpha < 0$

طبق دامنه لگاریتم $\rightarrow \sin\alpha > 0$

$\tan\alpha = -3 \xrightarrow{1+\tan^2 = \frac{1}{\cos^2}} 1 + (-3)^2 = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{1}{10}$

$\rightarrow \cos\alpha = \frac{1}{\sqrt{10}}$

$\xrightarrow{\sin^2\alpha + \cos^2\alpha = 1} \frac{1}{10} + \sin^2\alpha = 1 \rightarrow \sin\alpha = \frac{3}{\sqrt{10}}$

جواب: $\sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} - \frac{1}{\sqrt{10}} = \frac{2}{\sqrt{10}} = \frac{\sqrt{10}}{5}$

41)

$\sin^4\alpha + \cos^2\alpha = \frac{4}{5} \xrightarrow{\cos^2\alpha = 1 - \sin^2\alpha} \sin^4\alpha - \sin^2\alpha = \frac{-1}{5}$

$\sin^2\alpha(\sin^2\alpha - 1) = \frac{-1}{5} \rightarrow \sin^2\alpha(-\cos^2\alpha) = \frac{-1}{5}$

$\rightarrow \sin^2\alpha \cos^2\alpha = \frac{1}{5} \xrightarrow{\times 4} 4\sin^2\alpha\cos^2\alpha = \frac{4}{5} \rightarrow$

$(2\sin\alpha\cos\alpha)^2 = \frac{4}{5} \xrightarrow{2\sin\alpha\cos\alpha = \sin 2\alpha} (\sin 2\alpha)^2 = \frac{4}{5}$

$\cos^4\alpha + \sin^2\alpha = ? \xrightarrow{\sin^2\alpha = 1 - \cos^2\alpha} \cos^4\alpha + 1 - \cos^2\alpha =$

$\cos^2\alpha(\cos^2\alpha - 1) + 1 = \cos^2\alpha(-\sin^2\alpha) + 1 \rightarrow$

$\frac{-\cos^2\alpha\sin^2\alpha \times 4}{4} + 1 = \frac{-1}{4}(2\sin\alpha\cos\alpha)^2 + 1 \xrightarrow{2\sin\alpha\cos\alpha = \sin 2\alpha}$

$\frac{-\sin^2 2\alpha}{4} + 1 \xrightarrow{\sin^2 2\alpha = \frac{4}{5}} \frac{-\frac{4}{5}}{4} + 1 = \frac{4}{5}$

s.a.m

Subject: Date: / /

۵)

$$\frac{۳\sin x+\sin x\cos x}{۲-۲\cos^{۲}\alpha}<۰ \longrightarrow \frac{\sin\alpha(۳+\cos\alpha)}{۲(۱-\cos^{۲}\alpha)}<۰$$

$$\xrightarrow{۱-\cos^{۲}\alpha=\sin^{۲}\alpha} \frac{\sin\alpha\,\overbrace{(۳+\cos\alpha)}^{\text{همواره } \oplus \text{ (زیرا: } -۱<\cos<۱)}}{\underbrace{۲\sin^{۲}\alpha}_{\text{همواره } +}}<۰ \longrightarrow \sin\alpha<۰$$

$$\sin\alpha\tan\alpha-\frac{۱}{\cos\alpha}>۰ \longrightarrow \frac{\sin^{۲}\alpha}{\cos\alpha}-\frac{۱}{\cos\alpha}>۰ \longrightarrow \frac{\sin^{۲}\alpha-۱}{\cos\alpha}$$

$$\xrightarrow{۱-\sin^{۲}\alpha=\cos^{۲}\alpha} \frac{\overbrace{-\cos^{۲}\alpha}^{\text{همواره } \ominus \text{ است}}}{\cos\alpha}>۰ \longrightarrow \cos\alpha<۰$$

$$\left.\begin{matrix}\sin<۰\\ \cos\alpha<۰\end{matrix}\right\} \longrightarrow \text{ناحیه } ۳$$

۶)

$$\frac{۲}{\sin\alpha}+\frac{۳}{\cos\alpha}=۰ \longrightarrow \frac{۲\cos\alpha+۳\sin\alpha}{\sin\alpha\cos\alpha}=۰ \longrightarrow$$

$$۳\sin\alpha+۲\cos\alpha=۰ \longrightarrow ۳\sin\alpha=-۲\cos\alpha \longrightarrow$$

$$\frac{\sin\alpha}{\cos\alpha}=-\frac{۲}{۳} \longrightarrow \tan\alpha=\frac{-۲}{۳} \longrightarrow \cot\alpha=\frac{-۳}{۲}$$

$$\tan\alpha+\cot\alpha=-\frac{۲}{۳}-\frac{۳}{۲}=\frac{-۴-۹}{۶}=\frac{-۱۳}{۶}$$

s.a.m

Subject: Date: / /

71) $A=\dfrac{\sqrt{1+\sin 50}}{\sin 50+\sin 10}=\dfrac{\sqrt{1+\sin 50}}{\sin(60-10)+\sin 10}=\dfrac{\sqrt{1+\sin 50}}{\frac{\sqrt{3}}{2}\cos 10-\frac{1}{2}\sin 10+\sin 10}$

$\dfrac{\sqrt{1+\sin 50}}{\frac{1}{2}\sin 10+\frac{\sqrt{3}}{2}\cos 10}=\dfrac{\sqrt{1+\sin 50}}{\sin(70)}=\dfrac{\sqrt{1+\sin 50}}{\cos 20}$

$A^2=\dfrac{1+\sin 50}{\cos^2 20}\xrightarrow{\cos^2 20=\frac{1+\cos 40}{2}}\dfrac{(1+\sin 50)2}{1+\cos 40}=$

$\xrightarrow{\sin 50=\cos 40}\dfrac{2(1+\cos 40)}{1+\cos 40}=2$

---

81) $(1-2\sin^2 5)(2\cos^2 10-1)\left(\dfrac{1-\tan^2 20}{1+\tan^2 20}\right)=$

$\begin{cases}1-2\sin^2\alpha=\cos 2\alpha\\ 2\cos^2\alpha-1=\cos 2\alpha\\ 1+\tan^2\alpha=\dfrac{1}{\cos^2\alpha}\end{cases}\longrightarrow(\cos 10\times\cos 20)\left(\dfrac{1-\tan^2 20}{\frac{1}{\cos^2 20}}\right)$

$\rightarrow(\cos 10\cdot\cos 20)\left(\dfrac{\frac{\cos^2 20}{\cos^2 20}-\frac{\sin^2 20}{\cos^2 20}}{\frac{1}{\cos^2 20}}\right)=(\cos 10\times\cos 20)(\cos^2 20-\sin^2 20)$

$\cos 10\times\cos 20\times\cos 40=?\ \xrightarrow[\div 8\sin 10]{\times 8\sin 10}\ \dfrac{2\times2\times2\sin 10\times\cos 10\times\cos 20\times\cos 40}{8\sin 10}$

$\dfrac{2\times2\times\sin 20\times\cos 20\times\cos 40}{8\sin 10}=\dfrac{2\times\sin 40\times\cos 40}{8\sin 10}=\dfrac{\sin 80}{8\sin 10}=\dfrac{\cos 10}{8\sin 10}=\dfrac{\cot}{8}$

s.a.m

Subject: Date: / /

۹) $\sin\alpha + ۳\cos\alpha = ۲ \rightarrow \sin\alpha = ۲ - ۳\cos\alpha \rightarrow$

$\sin^۲\alpha + \cos^۲\alpha = ۱ \rightarrow (۲ - ۳\cos\alpha)^۲ + \cos^۲\alpha = ۱ \rightarrow ۱۰\cos^۲\alpha - ۱۲\cos\alpha + ۳ = ۰$

$۵\cos ۲\alpha - ۱۲\cos\alpha = ۵(۲\cos^۲\alpha - ۱) - ۱۲\cos\alpha \rightarrow$

$\underbrace{۱۰\cos^۲\alpha - ۱۲\cos\alpha} - ۵ = (-۳) - ۵ = -۸$

۱۰) $\sin\hat{B}\sin\hat{C} = \cos^۲\frac{\hat{A}}{۲}$ $\hat{B} = ۷۲^\circ$ $\hat{A} = ?$: $\triangle ABC$ در

$\sin\hat{B}\sin\hat{C} = \frac{۱}{۲}(\cos\hat{A} + ۱)$

$\downarrow$

$\cos(۱۸۰ - (\hat{B}+\hat{C})) = -\cos(B+C)$

$۲\sin\hat{B}\sin\hat{C} = -\cos B\cos C + \sin B\sin C + ۱ \rightarrow$

$\sin\hat{B}\sin\hat{C} + \cos B\cos C = ۱ \rightarrow \cos(C - B) = ۱ \rightarrow \hat{C} - \hat{B} = ۰$

$\hat{C} = \hat{B} \rightarrow \hat{B} = ۷۲$ $\hat{A} + \hat{B} + \hat{C} = ۱۸۰ \rightarrow \hat{A} + ۷۲ + ۷۲ = ۱۸۰$

$\hat{A} = ۳۶^\circ$