نام و نام خانوادگی ...کیانا ابراهیمی... پاسخنامه تشریحی تکلیف شماره .۱۵.. کلاس .دهم تجربی A...

۱۳٫۵

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$x_B = 0 \to y_B = \tan(0+\frac{\pi}{4}) = \tan\frac{\pi}{4} = 1 \quad B\,|\,{}^{0}_{1}$

$\tan x$ مجانب $x = \pm\frac{\pi}{2}$ دارد

$-2x + \frac{\pi}{4} = \frac{\pi}{2} \to x = -\frac{\pi}{8} \to x_A$

$-2x + \frac{\pi}{4} = -\frac{\pi}{2} \to x = \frac{3\pi}{8} \to x_B$

$S_{ABC} = \frac{1}{2}\left(\frac{3\pi}{8} - (-\frac{\pi}{8})\right) = \frac{\pi}{4}$

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$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{x=\frac{2}{3}} \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{\cos^2\alpha = 1-\sin^2\alpha}$

$\frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}(1-\sin^2\alpha) = 0 \xrightarrow{\times 36} 9\sin^2\alpha - 24\sin\alpha + 7 = 0 \to \sin\alpha = \frac{7}{3}$ ×

$\to \sin\alpha = \frac{1}{3} \to \cos\alpha = \frac{\sqrt{8}}{3}$

$\tan\alpha = \frac{\frac{1}{3}}{\frac{\sqrt{8}}{3}} = \frac{1}{\sqrt{8}} \to \quad 1+\tan^2\alpha = 1 + \frac{1}{8} = \frac{9}{8}$

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$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3 \to \log\frac{\sin\alpha}{-\cos\alpha} = \log(-\tan\alpha) = \log 3 \to \tan\alpha = -3$

$\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} = \frac{-6}{10} = -0/6 \qquad \sqrt{(\sin\alpha - \cos\alpha)^2} = \sqrt{\sin^2\alpha + \cos^2\alpha - \sin 2\alpha} = \sqrt{1-(-0/6)} = \sqrt{1/6}$

$= \frac{4}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \boxed{0/4\sqrt{10}}$

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$\sin^4\theta + \cos^2\theta = \frac{4}{5} \to (\sin^2\theta)^2 + \cos^2\theta = \frac{4}{5}$

$\to (1-\cos^2\theta)^2 + \cos^2\theta = \frac{4}{5} \to \cos^4\theta - \cos^2\theta = -\frac{1}{5}$

$\cos^4\theta - (1-\sin^2\theta) = -\frac{1}{5} \to \cos^4\theta + \sin^2\theta = \frac{4}{5}$

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$\frac{3\sin x + \sin x\cos x}{2 - 2\cos^2 x} < 0 \to \frac{\sin x\,(\overbrace{3+\cos x}^{\text{همواره } (+)})}{2\,(\underbrace{1-\cos^2 x}_{\sin^2 x})} < 0 \to \boxed{\sin x < 0}$

$\sin x\tan x - \frac{1}{\cos x} > 0 \to \frac{\sin^2 x - 1}{\cos x} > 0 \to \frac{-\cos^2 x}{\cos x} > 0 \to \boxed{\cos x < 0}$

ناحیه سوم

✓ ۲

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$\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \longrightarrow \frac{2}{\sin x} = \frac{-3}{\cos x} \longrightarrow 2\cos x = -3\sin x \xrightarrow{\div \cos x} 2 = -3\tan x$

$\tan x = -\frac{2}{3} \longrightarrow \cot x = -\frac{3}{2}$

$\tan x + \cot x = -\frac{2}{3} - \frac{3}{2} = \frac{-4-9}{6} = -\frac{13}{6}$

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$1 + \sin\alpha = \left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)^2 \qquad \sqrt{\sin 50^\circ + 1} = \sin 25 + \cos 25 = \sqrt{2}\sin 70 = \sqrt{2}\cos 20$

$\sin 50^\circ + \sin 10^\circ = \cancel{2}\,\cancel{\sin 30}\cos 20^\circ = \cos 20$

$\frac{\sqrt{1+\sin 50}}{\sin 50^\circ + \sin 10} = \frac{\sqrt{2}\,\cancel{\cos 20}}{\cancel{\cos 20}} = \sqrt{2}$

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$\underbrace{(1-2\sin^2 5^\circ)}_{\cos 10^\circ}\underbrace{(2\cos^2 10 - 1)}_{\cos 20}\underbrace{\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)}_{\cos 40^\circ} = \cos 10\cdot\cos 20\cdot\cos 40$

$\xrightarrow{\times\frac{\sin 10}{\sin 10}} \frac{1}{8}\times\frac{\sin 10\cdot\cos 10\cdot\cos 20\cdot\cos 40}{\sin 10} = \frac{\sin 80}{\sin 10} = \frac{\cos 10}{\sin 10} = \cancel{\tan 10} \quad \frac{1}{8}\cot 10^\circ$

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$\sin\alpha + 3\cos\alpha = 2 \longrightarrow \sin\alpha = 2 - \cos\alpha \qquad (\sin\alpha + 3\cos\alpha)^2 = \sin^2\alpha + 6\sin\alpha\cos\alpha + 9\cos^2\alpha = 4$

$(1-\cos^2\alpha) + 9\cos^2\alpha + 6(2-3\cos\alpha)\cos\alpha = 4$

$\Rightarrow -10\cos^2\alpha + 12\cos\alpha = 3$

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$5\cos 2\alpha - 12\cos\alpha = 5(\cos 2\alpha - 1) - 12\cos\alpha = \underbrace{10\cos^2\alpha - 12\cos\alpha}_{-3} - 5 = -8$

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$\cos^2\frac{A}{2} = \frac{1+\cos A}{2}$

$2\sin B\sin C = 1 + \cos A \quad \underrightarrow{A+B+C=\pi}$

$2\sin B\sin C = 1 + \cos(\pi-(B+C)) \rightarrow$

$2\sin B\sin C = 1 - \cos(B+C)$

$\sin B\sin C + \cos B\cos C = \cos(B-C) = 1$

$\downarrow$

$B = C$

$180^\circ - (72^\circ + 72^\circ) = 36^\circ$