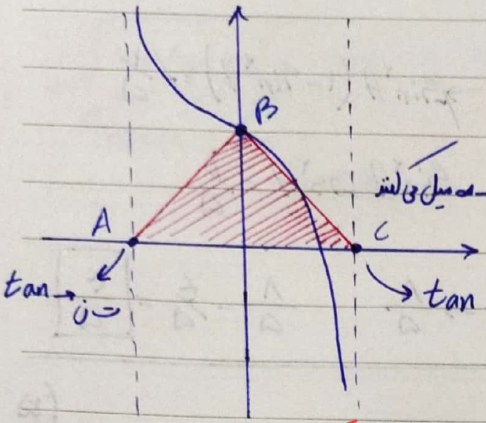
دوازدهم A — تکلیف ۱۵ — سارینا رضوی

۱۷/۲۵

۱)



B | 0 , 1   A | $-\frac{\pi}{8}$, 0   C | $\frac{3\pi}{8}$, 0

طبق نمودار tan در این ۲ نقطه تعریف نشده است و به $+\infty$ و $-\infty$ میل می‌کند

پس $-2x+\frac{\pi}{4}=\pm\frac{\pi}{2}$

$\to x=-\frac{\pi}{8}\to A$

$\to x=\frac{3\pi}{8}\to C$

$S=\frac{1}{2}\times\left(\frac{3\pi}{8}-\left(-\frac{\pi}{8}\right)\right)\times 1=\boxed{\frac{\pi}{4}}$

۲ ✓

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۲)

$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0 \to x^2-(\sin\alpha)x-\frac{1}{4}(1-\sin^2\alpha)=0$

$x=\frac{2}{3} \to \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}+\frac{1}{4}\sin^2\alpha=0$

$\to \frac{9\sin^2\alpha-24\sin\alpha+7}{36}=0 \to 9\sin^2\alpha-24\sin\alpha+7=0$

$\sin^2\alpha-24\sin\alpha+63=0 \to (\sin\alpha-21)(\sin\alpha-3)$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \to \boxed{\frac{9}{8}}$

$\sin\alpha \to \frac{1}{3}$ ✓ $\to \sin^2\alpha=\frac{1}{9}$, $\cos^2\alpha=\frac{8}{9}$

$\to \frac{21}{9} \to \frac{7}{3}$ غ ق ق

۲ ✓

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۳)

$\sin>0$, $\cos<0$

$\log^{\sin\alpha}-\log^{-\cos\alpha}=\log^{3}$

۲ ✓

$\log^{\frac{\sin\alpha}{-\cos\alpha}}=\log^{3} \to \frac{\sin\alpha}{-\cos\alpha}=3 \to \sin\alpha=-3\cos\alpha \xrightarrow{\text{توان ۲}} \underbrace{\sin^2\alpha}_{1-\cos^2\alpha}=9\cos^2\alpha$

$10\cos^2\alpha=1 \to \cos^2\alpha=\frac{1}{10} \to \cos\alpha$

$\to +\frac{\sqrt{10}}{10}$ ✗

$\to -\frac{\sqrt{10}}{10}$ ✓ $\to \sin\alpha=\frac{3\sqrt{10}}{10}$

$\sin\alpha-\cos\alpha=\frac{3\sqrt{10}+\sqrt{10}}{10}=\frac{2\sqrt{10}}{5}$

(۴

$\sin^4\theta+\cos^2\theta=\frac{۴}{۵} \longrightarrow \sin^4\theta-\sin^2\theta+1=\frac{۴}{۵}$

$\sin^4\theta-\sin^2\theta=-\frac{۱}{۵}$

$\cos^4\theta+\sin^2\theta=?$

$\sin^2\theta(\sin^2\theta-1)=-\frac{۱}{۵} \longrightarrow \sin^2\theta(1-\sin^2\theta)=\frac{۱}{۵}$

$\sin^4\theta+\cos^2\theta+\cos^4\theta+\sin^2\theta$ $\qquad \sin^2\theta\cos^2\theta=\frac{۱}{۵}$

$\hookrightarrow 1+(\sin^2\theta+\cos^2\theta)^۲-۲\sin^2\theta\cos^2\theta \longrightarrow ۲-\frac{۲}{۵} \to \frac{۸}{۵} \qquad \frac{۸}{۵}-\frac{۴}{۵}=\boxed{\frac{۴}{۵}}$

۲ ✓

(۵

$\frac{۳\sin x+\sin x\cos x}{۲-۲\cos^2 x}<0 \qquad \sin x\tan x-\frac{1}{\cos x}>0$

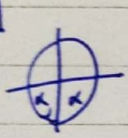
$\frac{\sin x(۳+\cos x)}{۲(1-\cos^2 x)}<0$ $\quad (۳+\cos x) \to$ همواره (+) $\quad 1-\cos^2x = \sin^2 x \to$ همواره (+)

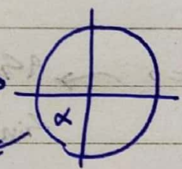
$\boxed{\sin x<0}$

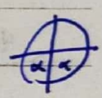
$\frac{\sin^2 x}{\cos x}-\frac{1}{\cos x}>0$

$\frac{\sin^2 x-1}{\cos x}>0 \longrightarrow \frac{-\cos^2 x}{\cos x}>0 \quad (-) \quad \to \cos x<0$

۲ ✓

$\mathrm{I} \to$ 

$\mathrm{I}\cap\mathrm{II} \to$  ← ناحیه سوم

$\mathrm{II} \to$ 

(۶

$\frac{۲}{\sin x}+\frac{۳}{\cos x}=0 \longrightarrow \frac{۲\cos x+۳\sin x}{\sin x\cos x}=0 \longrightarrow ۲\cos x+۳\sin x=0$

$\sin x=-\frac{۲}{۳}\cos x$ ← غیر هم علامت

$\tan x+\cot x \to \frac{\sin^2 x+\cos^2 x}{\sin x\cos x} \to \frac{1}{\sin x\cos x} \to \frac{1}{\frac{-۶}{۱۳}}=\boxed{-\frac{۱۳}{۶}}$

$\cos^2 x+\sin^2 x=1 \to \cos^2 x+\frac{۴}{۹}\cos^2 x=1 \longrightarrow \cos^2 x=\frac{۹}{۱۳} \qquad \sin^2 x=\frac{۴}{۱۳}$

$|\sin x\cos x|=\sqrt{\cos^2 x\sin^2 x}=\sqrt{\frac{۹}{۱۳}\times\frac{۴}{۱۳}} \to$

۲ ✓

$\frac{-۶}{۱۳}$ ← غیر هم علامت $\cos x$, $\sin x$ ← $\frac{۶}{۱۳}$

$$\sqrt{1+\cos 40} = \sqrt{2\cos^2 20} = \sqrt{2}\cos 20$$

7)

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ + \sin 10^\circ} \rightarrow 1+2\sin 25\cos 25 \rightarrow \sqrt{(\sin 25+\cos 25)^2} = \sin 25+\cos 25$$

$$\sin 25+\cos 25 = \sqrt{2}\sin(25+45)$$

$$\sin(20+30)+\sin(30-10) \rightarrow \sqrt{2}\sin 70 \quad \frac{\sqrt{2}\cos 20}{2\sin 30\cos 20} = \sqrt{2}$$

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8)

$$(1-2\sin^2 5)(2\cos^2 10-1)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right) \rightarrow \cos 10\times\cos 20\times\cos 40 = ?$$

$(1-\cos 10)$ $(1+\cos 20)$ $\cos 40$

$$\xrightarrow[\div \sin 10]{\times \sin 10} \frac{1}{8}\,\frac{\sin 80}{\sin 10} \rightarrow \cos 10$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \qquad \cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$2\cos^2\alpha = 1+\cos 2\alpha$$

$$1-\cos 2\alpha = 2\sin^2\alpha$$

$$\rightarrow \boxed{\frac{1}{8}\cot 10^\circ}$$

2

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9)

$$\sin\alpha + 3\cos\alpha = 2 \qquad 5\cos 2\alpha - 12\cos\alpha$$

$$\cos 2\alpha \rightarrow \cos^2\alpha-\sin^2\alpha \rightarrow \cos^2\alpha-(1-\cos^2\alpha) \rightarrow 2\cos^2\alpha - 1$$

$$\underbrace{10\cos^2\alpha - 12\cos\alpha}_{-3} - 5 = ? \rightarrow \boxed{-8}$$

$$\sin^2\alpha+\cos^2\alpha = 1 \Rightarrow (2-3\cos\alpha)^2+\cos^2\alpha = 1 \rightarrow 10\cos^2\alpha - 12\cos\alpha + 4 = 1$$

$$10\cos^2\alpha - 12\cos\alpha = -3$$

2

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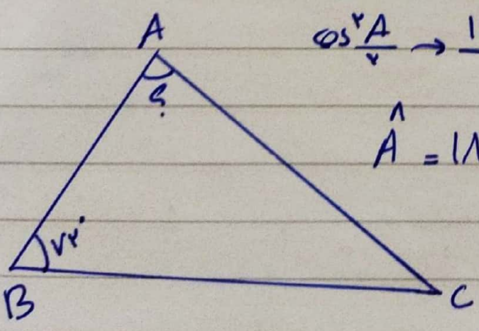
10)



$$\cos^2\frac{A}{2} \rightarrow \frac{1+\cos A}{2} = \sin\hat{B}\sin\hat{C} \rightarrow 1+\cos\hat{A} = 2\sin\hat{B}\sin\hat{C}$$

$$\hat{A} = 180 - \hat{B} - \hat{C} \rightarrow \hat{A} = \pi - (\hat{B}+\hat{C})$$

$$\cos\hat{A} = \cos(\pi-(B+C)) \rightarrow \cos A = -\cos(B+C)$$

$$1 - \underbrace{\cos(\hat{B}+\hat{C})}_{\cos B\cos C - \sin B\sin C} = 2\sin\hat{B}\sin\hat{C}$$

1.5

$$\sin B\sin C + \cos B\cos C = 1$$

$$\cos(B-C) = 1 \longrightarrow B = C$$

$$180 - (72+72) = 36$$