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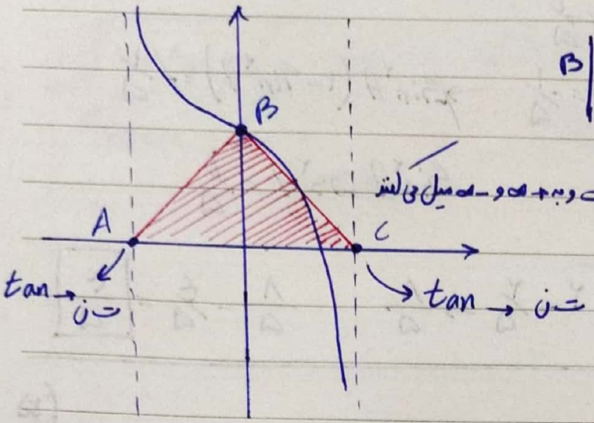
دوازدهم A — تکلیف ۱۵ — سارینا رضوی



(۱ $C\left|\begin{matrix}\frac{3\pi}{8}\\0\end{matrix}\right.$ $A\left|\begin{matrix}\frac{-\pi}{8}\\0\end{matrix}\right.$ $B\left|\begin{matrix}0\\1\end{matrix}\right.$

طبق نمودار tan در این ۲ نقطه تعریف نشده است و به $+\infty$ و $-\infty$ میل می‌کند

پس $-2x+\frac{\pi}{4}=\pm\frac{\pi}{2}$

$\rightarrow x=-\frac{\pi}{8}\rightarrow A$

$\rightarrow x=\frac{3\pi}{8}\rightarrow C$

$$S=\frac{1}{2}\times\left(\frac{3\pi}{8}-\left(-\frac{\pi}{8}\right)\right)\times 1=\boxed{\frac{\pi}{4}}$$

(۲

$$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0\rightarrow x^2-(\sin\alpha)x-\frac{1}{4}(1-\sin^2\alpha)=0$$

$$x=\frac{2}{3}\rightarrow\frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}+\frac{1}{4}\sin^2\alpha=0$$

$$\rightarrow\frac{9\sin^2\alpha-24\sin\alpha+7}{36}=0\rightarrow 9\sin^2\alpha-24\sin\alpha+7=0$$

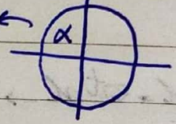
$$\sin^2\alpha-24\sin\alpha+63=0\rightarrow(\sin\alpha-21)(\sin\alpha-3)$$

$\sin\alpha\rightarrow\frac{1}{3}$ ✓ $\rightarrow\sin^2\alpha=\frac{1}{9}$ و $\cos^2\alpha=\frac{8}{9}$

$\rightarrow\frac{21}{9}\rightarrow\frac{7}{3}$ غ ق ق

$$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\rightarrow\boxed{\frac{9}{8}}$$

(۳



$\sin>0$ ، $\cos<0$

$$\log^{\sin\alpha}-\log^{-\cos\alpha}=\log^{3}$$

$$\log^{\frac{\sin\alpha}{-\cos\alpha}}=\log^{3}\rightarrow\frac{\sin\alpha}{-\cos\alpha}=3\rightarrow\sin\alpha=-3\cos\alpha\xrightarrow{\text{توان ۲}}\sin^2\alpha=9\cos^2\alpha$$

$\sin^2\alpha\downarrow 1-\cos^2\alpha$

$$10\cos^2\alpha=1\rightarrow\cos^2\alpha=\frac{1}{10}\rightarrow\cos\alpha$$

$\rightarrow+\frac{\sqrt{10}}{10}$ ✗

$\rightarrow-\frac{\sqrt{10}}{10}$ ✓ $\rightarrow\sin\alpha=\frac{3\sqrt{10}}{10}$

$$\sin\alpha-\cos\alpha=\frac{3\sqrt{10}+\sqrt{10}}{10}=\frac{2\sqrt{10}}{5}$$

۴)

$\sin^4\theta + \cos^2\theta = \frac{۴}{۵} \longrightarrow \sin^4\theta - \sin^2\theta + ۱ = \frac{۴}{۵}$

$\sin^4\theta - \sin^2\theta = -\frac{۱}{۵}$

$\cos^4\theta + \sin^2\theta = ?$

$\sin^2\theta(\sin^2\theta - ۱) = -\frac{۱}{۵} \longrightarrow -\sin^2\theta(۱ - \sin^2\theta) = -\frac{۱}{۵}$

$\sin^4\theta + \cos^2\theta + \cos^4\theta + \sin^2\theta$

$\sin^2\theta \cos^2\theta = \frac{۱}{۵}$

$\hookrightarrow ۱ + (\sin^2\theta + \cos^2\theta)^2 - ۲\sin^2\theta\cos^2\theta \longrightarrow ۲ - \frac{۲}{۵} \to \frac{۸}{۵}$

$\frac{۸}{۵} - \frac{۴}{۵} = \boxed{\frac{۴}{۵}}$

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۵)

$\frac{۳\sin x + \sin x\cos x}{۲ - ۲\cos^2 x} < ۰$

$\sin x\tan x - \frac{۱}{\cos x} > ۰$

$\frac{\sin x(۳ + \cos x)}{۲(۱ - \cos^2 x)} < ۰$

$۳ + \cos x \to$ همواره $(+)$

$۱ - \cos^2 x = \sin^2 x \to$ همواره $(+)$

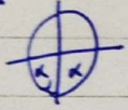
$\boxed{\sin x < ۰}$

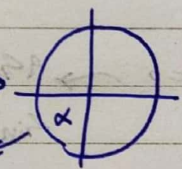
$\frac{\sin^2 x}{\cos x} - \frac{۱}{\cos x} > ۰$

$\frac{\sin^2 x - ۱}{\cos x} > ۰ \longrightarrow \frac{-\cos^2 x}{\cos x} > ۰$

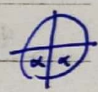
$-\cos^2 x \to (-)$

$\cos x < ۰$

$\mathrm{I} \to$ 

$\mathrm{I} \cap \mathrm{II} \to$ 

ناحیه سوم

$\mathrm{II} \to$ 

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۶)

$\frac{۲}{\sin x} + \frac{۳}{\cos x} = ۰ \longrightarrow \frac{۲\cos x + ۳\sin x}{\sin x\cos x} = ۰ \longrightarrow ۲\cos x + ۳\sin x = ۰$

$\sin x = -\frac{۲}{۳}\cos x$

غیر هم علامت

$\tan x + \cot x \to \frac{\sin^2 x + \cos^2 x}{\sin x\cos x} \to \frac{۱}{\sin x\cos x} \to \frac{۱}{\frac{-۶}{۱۳}} = \boxed{-\frac{۱۳}{۶}}$

$\cos^2 x + \sin^2 x = ۱ \to \cos^2 x + \frac{۴}{۹}\cos^2 x = ۱ \to \cos^2 x = \frac{۹}{۱۳}$ $\quad \sin^2 x = \frac{۴}{۱۳}$

$|\sin x\cos x| = \sqrt{\cos^2 x\sin^2 x} = \sqrt{\frac{۹}{۱۳} \times \frac{۴}{۱۳}} \to$

$\frac{۶}{۱۳} \to \sin x$ و $\cos x$ غیر هم علامت $\to \frac{-۶}{۱۳}$

۷)

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ + \sin 10^\circ} \longrightarrow 1+2\sin 25\cos 25 \rightarrow \sqrt{(\sin 25+\cos 25)^2} = \sin 25+\cos 25$$

$$\sin 25+\cos 25 = \sqrt{2}\sin(25+45)$$

$$\rightarrow \sqrt{2}\sin 70$$

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۸)

$$\left(1-2\sin^2 5^\circ\right)\left(2\cos^2 10-1\right)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right) \rightarrow \cos 10\times\cos 20\times\cos 40 = ?$$

$(1-\cos 10)$ $\quad(1+\cos 20)\quad \cos 40$

$$\xrightarrow[\div \sin 10]{\times \sin 10} \frac{1}{8}\,\frac{\sin 80}{\sin 10} \rightarrow \cos 10$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \qquad \cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$2\cos^2\alpha = 1+\cos 2\alpha$$

$$1-\cos 2\alpha = 2\sin^2\alpha$$

$$\longrightarrow \boxed{\frac{1}{8}\cot 10^\circ}$$

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۹)

$$\sin\alpha + 3\cos\alpha = 2$$

$$5\cos 2\alpha - 12\cos\alpha$$

$$\hookrightarrow \cos^2\alpha - \sin^2\alpha \rightarrow \cos^2\alpha - (1-\cos^2\alpha) \rightarrow 2\cos^2\alpha - 1$$

$$\cancel{10\cos^2\alpha - 12\cos\alpha}_{-3} - 5 = ? \rightarrow \boxed{-8}$$

$$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (2-3\cos\alpha)^2 + \cos^2\alpha = 1 \rightarrow 10\cos^2\alpha - 12\cos\alpha + 4 = 1$$

$$10\cos^2\alpha - 12\cos\alpha = -3$$

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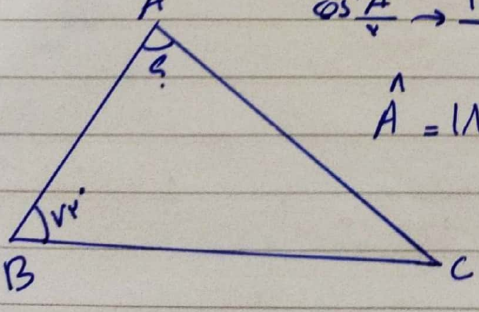
۱۰)



$$\cos^2\frac{A}{2} \rightarrow \frac{1+\cos A}{2} = \sin\hat{B}\sin\hat{C} \rightarrow 1+\cos\hat{A} = 2\sin\hat{B}\sin\hat{C}$$

$$\hat{A} = 180 - \hat{B} - \hat{C} \rightarrow \hat{A} = \pi - (\hat{B}+\hat{C})$$

$$\cos\hat{A} = \cos\left(\pi - (B+C)\right) \rightarrow \cos A = -\cos(B+C)$$

$$1 - \underbrace{\cos(\hat{B}+\hat{C})}_{\cos B\cos C - \sin B\sin C} = 2\sin\hat{B}\sin\hat{C}$$