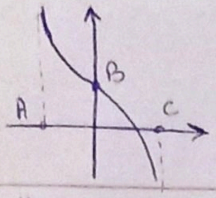
دوازدهم دفتر A - جمعه‌ها | تکلیف شماره ۱۵ | سپیده ثنایی طبا

1-



$\tan(-2x+\frac{\pi}{4})$ $\quad AC=T=\frac{\pi}{|b|}=\frac{\pi}{2}$

$f(0)=\tan(\frac{\pi}{4})=1=B$ $\quad S=\frac{1}{2}\times AC\times OB=\frac{1}{2}\times\frac{\pi}{2}\times 1=\frac{\pi}{4}$

2-

$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0$ $\quad x=\frac{2}{3}$ $\quad 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\frac{1}{\frac{8}{9}}=\frac{9}{8}$

$\Rightarrow \frac{4}{9}-\frac{2}{3}(\sin\alpha)-\frac{1}{4}\cos^2\alpha=0 \Rightarrow \frac{4}{9}-\frac{2}{3}(\sin\alpha)-\frac{1}{4}(1-\sin^2\alpha)=0$

$\Rightarrow \frac{1}{4}\sin^2\alpha-\frac{2}{3}\sin\alpha+\underbrace{\frac{4}{9}-\frac{1}{4}}_{\frac{16-9}{36}}=0 \Rightarrow 9\sin^2\alpha-24\sin\alpha+7=0 \Rightarrow (3\sin\alpha-1)(3\sin\alpha-7)=0$

$\sin\alpha=\frac{1}{3}$ ق ق

$\sin\alpha=\frac{7}{3}$ غ ق ق

$\underbrace{\sin^2\alpha}_{\frac{1}{9}}+\cos^2\alpha=1 \Rightarrow \cos^2\alpha=\frac{8}{9}$

3-

$\log(\sin\alpha)-\log(-\cos\alpha)=\log 3$ $\quad \log_b a$: $a>0$, $b>0$, $b\neq 1$ $\Rightarrow \sin\alpha>0$, $\cos\alpha<0$

$\log\frac{\sin\alpha}{-\cos\alpha}=\log 3 \Rightarrow \frac{\sin\alpha}{-\cos\alpha}=3 \Rightarrow \sin\alpha=-3\cos\alpha$, $\sin^2\alpha=9\cos^2\alpha$ $\quad \sin^2\alpha+\cos^2\alpha=1 \Rightarrow 10\cos^2\alpha=1$

$\Rightarrow \cos\alpha=\frac{-\sqrt{10}}{10}$, $\sin\alpha=\frac{\sqrt{90}}{10}$

$\sin\alpha-\cos\alpha=\frac{\sqrt{90}}{10}+\frac{\sqrt{10}}{10}=\frac{3\sqrt{10}+\sqrt{10}}{10}=\frac{4\sqrt{10}}{10}=\frac{2\sqrt{10}}{5}$

4-

$\sin^4\alpha+\cos^2\alpha=\frac{4}{5} \Rightarrow \sin^4\alpha+1-\sin^2\alpha=\frac{4}{5} \Rightarrow \sin^4\alpha-\sin^2\alpha+\frac{1}{5}=0$

$t^2-t+\frac{1}{5}=0$ $\quad t=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{1\pm\frac{\sqrt{5}}{5}}{2}$

$\sin^2\alpha=\frac{5+\sqrt{5}}{10}$ ق ق $\rightarrow \cos^2\alpha=\frac{5-\sqrt{5}}{10}$

$\sin^2\alpha=\frac{5-\sqrt{5}}{10}$ ق ق $\rightarrow \cos^2\alpha=\frac{5+\sqrt{5}}{10}$

$\cos^4\alpha+\sin^2\alpha \xrightarrow{\text{حالت ۱}} \frac{25+5-10\sqrt{5}}{100}+\frac{5+\sqrt{5}}{10}=0/8$

حالت ۲ $\rightarrow \frac{25+5+10\sqrt{5}}{100}+\frac{5-\sqrt{5}}{10}=0/8$

$\Rightarrow \cos^4\alpha+\sin^2\alpha=0/8$

5-

$\frac{3\sin\alpha+\sin\alpha\cos\alpha}{2-2\cos^2\alpha}<0 \Rightarrow \frac{\sin\alpha(3+\cos\alpha)}{2(1-\cos^2\alpha)}<0 \Rightarrow \frac{\sin\alpha(\overbrace{3+\cos\alpha}^{\text{همواره}+})}{\underbrace{2\sin^2\alpha}_{\text{همواره}+}}<0 \Rightarrow \sin\alpha<0$

ناحیه سوم

$\sin\alpha\tan\alpha-\frac{1}{\cos\alpha}>0 \Rightarrow \frac{\sin^2\alpha}{\cos\alpha}-\frac{1}{\cos\alpha}>0 \Rightarrow \frac{-\cos^2\alpha}{\cos\alpha}>0 \Rightarrow -\cos\alpha>0 \Rightarrow \cos\alpha<0$

6-

$\frac{2}{\sin x}=\frac{-3}{\cos x} \Rightarrow 2\cos x=-3\sin x \Rightarrow \cos x=\frac{-3}{2}\sin x$

$\sin^2x+\cos^2x=1 \Rightarrow \sin^2x+\frac{9}{4}\sin^2x=1 \Rightarrow \sin^2x=\frac{4}{13}$, $\cos^2x=\frac{9}{13}$ $\quad \sin x=\frac{2}{\sqrt{13}}$ $\quad \cos x=\frac{-3}{\sqrt{13}}$

$\tan x+\cot x=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{1}{\sin x\cos x}=\frac{1}{\frac{2}{\sqrt{13}}\times\frac{-3}{\sqrt{13}}}=\frac{-13}{6}$

-7

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ+\sin 10^\circ}=\frac{\sqrt{1+2\sin 25^\circ\cos 25^\circ}}{\sin 50^\circ+\sin 10^\circ}=\frac{\sqrt{\sin^2 25^\circ+\cos^2 25^\circ+2\sin 25^\circ\cos 25^\circ}}{\sin 50^\circ+\sin 10^\circ}=\frac{|\sin 25^\circ+\cos 25^\circ|}{\sin 50^\circ+\sin 10^\circ}$$

$\sin 50^\circ=2\sin 25^\circ\cos 25^\circ$ $\quad$ $\sin 50^\circ=\sin(30+20)=\sin 30^\circ\cos 20^\circ+\cos 30^\circ\sin 20^\circ$

$\sin 10^\circ=\sin(30-20)=\sin 30^\circ\cos 20^\circ-\cos 30^\circ\sin 20^\circ$

$\sin 50^\circ+\sin 10^\circ=2\underbrace{\sin 30^\circ}_{\frac{1}{2}}\cos 20=\cos 20^\circ$

2 ✓

$\sin 25^\circ+\cos 25^\circ=\sqrt{2}\sin(25^\circ+45^\circ)=\sqrt{2}\sin 70^\circ=\sqrt{2}\cos 20^\circ$ $\quad$ $\frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ}=\sqrt{2}$

-8

$$\underbrace{(1-2\sin^2 5^\circ)}_{\cos 10^\circ}\underbrace{(2\cos^2 10^\circ-1)}_{\cos 20^\circ}\underbrace{\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right)}_{\cos 40^\circ}\times\frac{\sin 10^\circ}{\sin 10^\circ}=\frac{1}{2}\sin 20^\circ\cos 20^\circ\cos 40^\circ=$$

$\sin\alpha=\cos\left(\frac{\pi}{2}-\alpha\right)$

$\frac{1}{4}\sin 40^\circ\cos 40^\circ=\frac{1}{8}\sin 80^\circ=\frac{\sin 80^\circ}{8\times\sin 10^\circ}$ $\quad$ $\frac{\cos 10^\circ}{8\times\sin 10^\circ}=\underline{\frac{1}{8}\cot 10^\circ}$

2 ✓

-9

$\sin\alpha+3\cos\alpha=2\rightarrow\sin\alpha=2-3\cos\alpha$

$\cos^2\alpha+\sin^2\alpha=1\Rightarrow\cos^2\alpha+9\cos^2\alpha-12\cos\alpha+4=1\Rightarrow$

$10\cos^2\alpha-12\cos\alpha+3=0\Rightarrow\cos\alpha=\frac{12\pm2\sqrt{6}}{20}=\frac{3\pm\sqrt{6}}{5}$ $\quad$ $10\cos^2\alpha-12\cos\alpha=-3$

$\cos 2\alpha=2\cos^2\alpha-1$ $\quad$ $5\cos 2\alpha-12\cos\alpha=\underbrace{10\cos^2\alpha-12\cos\alpha}_{-3}-5=\underline{-8}$

2 ✓

-10

$\cos^2\frac{A}{2}=\frac{1+\cos A}{2}$

0

$2\sin\hat{B}\sin\hat{C}=1+\cos\hat{A}$ $\quad$ $\xrightarrow{A+B+C=\pi}$ $\hat{A}=\pi-(B+C)$

$2\sin\hat{B}\sin\hat{C}=1+\cos(\pi-(B+C))\rightarrow 2\sin\hat{B}\sin\hat{C}=1-\cos(\hat{B}+\hat{C})$

$2\sin B\sin C=1-(\cos\hat{B}\cos\hat{C}-\sin B\sin C)=$

$\sin B\sin C+\cos B\cos C=1\rightarrow\cos(\hat{B}-\hat{C})=1\rightarrow B=C$

$180-(72+72)=36^\circ$