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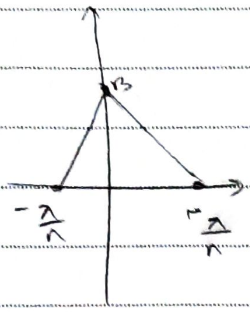

$y = \tan(-2x + \frac{\pi}{4})$

$B: x = 0 \to \tan\frac{\pi}{4} = 1 \to B(0,1)$

$A, C: \tan(-2x + \frac{\pi}{4}) \to$ تن

$\Rightarrow \tan(-2x + \frac{\pi}{4}) = \tan\frac{\pi}{2} \to x = -\frac{\pi}{8}$

$\tan(-2x + \frac{\pi}{4}) = \tan(-\frac{\pi}{2}) \to x = \frac{3\pi}{8}$

$\frac{1 \times (\frac{3\pi}{8} - (-\frac{\pi}{8}))}{2} = \boxed{\frac{\pi}{4}}$

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$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{x = \frac{2}{3}} \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}(1 - \sin^2\alpha) = 0$

$= 9\sin^2\alpha - 24\sin\alpha + 7 = 0 \qquad \sin\alpha = \frac{24 \pm \sqrt{576 - 252}}{18} = \frac{24 \pm 18}{18} \Rightarrow \frac{42}{18}$ غ ق ق

$\frac{6}{18} = \frac{1}{3}$ ق ق

$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{1}{9} = \frac{8}{9}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \boxed{\frac{9}{8}}$

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۳-

$\log^{\sin\alpha} - \log^{-\cos\alpha} = \log^{3} \Rightarrow \log^{\frac{\sin\alpha}{-\cos\alpha}} = \log^{3} \Rightarrow \sin\alpha = -3\cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow 9\cos^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{1}{10} \Rightarrow \cos\alpha = -\frac{\sqrt{10}}{10} \qquad \sin\alpha = \frac{3\sqrt{10}}{10}$

$\sin\alpha - \cos\alpha = \frac{3\sqrt{10}}{10} - (-\frac{\sqrt{10}}{10}) = \frac{4\sqrt{10}}{10} = \boxed{\frac{2\sqrt{10}}{5}}$

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$\sin^4\theta + \cos^2\theta = \frac{4}{5} \Rightarrow \sin^4\theta + 1 - \sin^2\theta = \frac{4}{5} \Rightarrow \sin^4\theta - \sin^2\theta = -\frac{1}{5}$

$\sin^2\theta(\sin^2\theta - 1) = \frac{1}{5} \Rightarrow \sin^2\theta\cos^2\theta = \frac{1}{5}$

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$\cos^4\alpha + \sin^2\alpha = \cos^4\theta + 1 - \cos^2\alpha = \cos^2(\cos^2 - 1) + 1 = -\sin^2\theta\cos^2\theta + 1 = -\frac{1}{5} + 1 = \boxed{\frac{4}{5}}$

۵-

$\frac{3\sin x + \sin x\cos x}{2 - 2\cos^2 x} < 0 \Rightarrow \frac{\sin x(3 + \cos x)}{2(\underbrace{1 - \cos^2 x}_{\sin^2 x})} = \frac{\overbrace{3 + \cos x}^{+}}{2\sin x} < 0$

$\Rightarrow 2\sin x < 0 \Rightarrow \sin x < 0 \Rightarrow$

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$\sin x\tan x - \frac{1}{\cos x} > 0 \Rightarrow \frac{1}{\cos x}(\underbrace{\sin^2 x - 1}_{-\cos^2 x}) > 0 \Rightarrow -\cos x > 0 \Rightarrow \cos x < 0 \Rightarrow$

$\Rightarrow$ $\boxed{\text{ناحیه ۳}}$

۶ —
$$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \Rightarrow \frac{2\cos x+3\sin x}{\sin x\cos x}=0 \to 2\cos x+3\sin x=0$$

$$\Rightarrow \cos x=-\frac{3}{2}\sin x \qquad \sin^2 x+\cos^2 x=1 \Rightarrow \sin^2 x+\frac{9}{4}\sin^2 x=1$$

$$\Rightarrow \sin^2 x=\frac{4}{13} \Rightarrow \sin x=\pm\frac{2}{\sqrt{13}}=\pm\frac{2\sqrt{13}}{13} \Rightarrow \cos x=\pm\frac{3\sqrt{13}}{13}$$ مختلف العلامت اند

$$\tan x+\cot x=\frac{\sin^2+\cos^2}{\sin\cos}=\frac{1}{\sin\cos}=\frac{1}{-\frac{2\sqrt{13}}{13}\times\frac{3\sqrt{13}}{13}}=\boxed{-\frac{13}{6}}$$

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$$1+\sin 50=1+2\sin 25\cos 25=(\sin 25+\cos 25)^2 \Rightarrow \sqrt{1+\sin 50}=\sin 25+\cos 25$$ ✓

$$=\sqrt{2}\sin(25+45)=\sqrt{2}\sin 70=\sqrt{2}\cos 20$$

$$\sin 50+\sin 10=\sin(20+30)+\sin(30-20)=\cancel{\sin 20\cos 30}+\sin 30\cos 20+\sin 30\cos 20$$

$$\cancel{-\sin 20\cos 30}=2\underbrace{\sin 30}_{\frac{1}{2}}\cos 20=\cos 20$$

$$\boxed{\frac{\sqrt{2}\cos 20}{\cos 20}=\sqrt{2}}$$

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۸ —
$$\frac{1-\tan^2 20}{1+\tan^2 20}=\cos 40 \qquad \cos 10\times\cos 20\times\cos 40 \xrightarrow[\div\sin 10]{\times\sin 10}$$

$$=\frac{1}{2}\sin 20=\frac{1}{4}\sin 40=\frac{1}{8}\frac{\sin 80}{\sin 10}=\frac{1}{8}\frac{\sin(\frac{\pi}{2}-10)}{\sin 10}=\boxed{\frac{1}{8}\cot 10}$$

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۹ —
$$\cos 2\alpha=2\cos^2\alpha-1 \Rightarrow 5\cos 2\alpha-12\cos\alpha=10\cos^2\alpha-12\cos\alpha-5$$

$$\sin\alpha+3\cos\alpha=2 \Rightarrow \sin\alpha=2-3\cos\alpha \Rightarrow \sin^2\alpha+\cos^2\alpha=1$$

$$(2-3\cos\alpha)^2+\cos^2\alpha=9\cos^2\alpha+4-12\cos\alpha+\cos^2\alpha=10\cos^2\alpha-12\cos\alpha+4=1$$

$$10\cos^2\alpha-12\cos\alpha=-3 \Rightarrow 10\cos^2\alpha-12\cos\alpha-5=-3-5=\boxed{-8}$$

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۱۰ —
$$\cos^2\frac{A}{2}=\frac{1+\cos A}{2} \qquad \cos A=\cos(\pi-(B+C))=-\cos(B+C)$$

$$\Rightarrow 1-\cos(B+C)=2\sin B\sin C \qquad 1-(\cos B\cos C-\sin B\sin C)=2\sin B\sin C$$

$$1-\cos B\cos C+\sin B\sin C=2\sin B\sin C$$

$$1-(\cos B\cos C+\sin B\sin C)=0 \Rightarrow 1-\cos(B-C)=0 \Rightarrow \cos(B-C)=1$$

$$\Rightarrow B-C=0 \Rightarrow B=C=72 \qquad A=180-(2\times 72)=\boxed{36}$$

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