۱۵/۵

نرگس زندمحل

۱) الف) $\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow \cos 2x = 1-2\sin^2 x$

$\Rightarrow \underbrace{1-2\sin^2 x - 3\sin x + 1}_{} = 0 \Rightarrow \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{+3\pm\sqrt{9+16}}{-4} = \frac{3\pm 5}{-4} \to -2$, $\to \frac{1}{2}$ ✓

$\sin x = \frac{1}{2} \Rightarrow 2k\pi + \frac{\pi}{6} \mid 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6}$, $x = \{2k\pi + \frac{\pi}{6}$ و $2k\pi + \frac{5\pi}{6}\}$

ب) $\cos^2 x = \frac{1+\cos 2x}{2} \Rightarrow 2\cos^2 x - 1 = \cos 2x \Rightarrow 2\cos^2 x + \cos x - 2 = 0$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm 3}{4} \to -\frac{1}{2}$ × , $\to 1 \Rightarrow \cos x = 1 \Rightarrow x = \{2k\pi\}$

سوال ۲ الف)

$\sin^4 x - \cos^4 x = \sin 4x \rightarrow -(\cos^2 x - \sin^2 x) = \sin 4x$

$-\cos 2x = \sin 4x \rightarrow \sin 4x + \cos 2x = 0 \rightarrow 2\sin 2x \cos 2x + \cos 2x = 0$

$\cos 2x (2\sin 2x + 1) = 0 \rightarrow \cos 2x = 0 \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\rightarrow \sin 2x = -\frac{1}{2} \rightarrow x = k\pi - \frac{\pi}{12}$ ، $k\pi + \frac{7\pi}{12}$

1,25 ۲)

الف) $\sin^4 x - \cos^4 x = 1 \times (\sin^2 x - \cos^2 x) = 2\sin 2x \cos 2x$

$\Rightarrow \sin^2 x - \cos^2 x = 4\sin x \cos x (\cos^2 x - \sin^2 x) \Rightarrow \cos 2x = -\frac{1}{4}\sin x$

$\Rightarrow x = -\frac{1}{4}(2k\pi + x) \Rightarrow x = -\frac{1}{4}x - \frac{k\pi}{2} \Rightarrow \frac{5}{4}x = -\frac{k\pi}{2} \Rightarrow x = -\frac{2k\pi}{5}$

$x = -\frac{1}{4}(2k\pi + \pi - x) \Rightarrow x = -\frac{k\pi}{2} - \frac{\pi}{4} + \frac{1}{4}x \Rightarrow \frac{3}{4}x = -\frac{k\pi}{2} - \frac{\pi}{4} \Rightarrow x = -\frac{2k\pi}{3} - \frac{\pi}{3}$

ب) $2\cos^2 x - 1 = -2\sin x \cos x \Rightarrow \cos^2 x - \sin^2 x = -2\sin x \cos x \Rightarrow \cos 2x = -\sin 2x$

$-2x = 2k\pi + 2x \Rightarrow -4x = 2k\pi \Rightarrow x = -\frac{k\pi}{2}$

۳)

الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \rightarrow \sin x \neq 0 \Rightarrow \cos x(2\cos x + 1) = 1$

$\Rightarrow 2\cos^2 x + \cos x - 1 = 0 \rightarrow \cos x = -1 \rightarrow 2k\pi + \pi$

$\rightarrow -\frac{c}{a} = +\frac{1}{2} \rightarrow 2k\pi \pm \frac{\pi}{3}$

1,5

سوال ۲ قسمت ب

$2\cos^2 x + 2\sin x \cos x = 1$

$\begin{cases} 2\sin x \cos x = \sin 2x \\ \frac{1+\cos 2x}{2} = \cos^2 x \end{cases}$

$1 + \cos 2x + \sin 2x = 1 \rightarrow \cos 2x + \sin 2x = 0$

$\tan 2x = -1 \rightarrow 2x = k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

$\cos x + \sin x = 0 \rightarrow \tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}$

۳) ب) $2 - 2\sin^2 x = 2\cos^2 x \Rightarrow 2\cos^2 x = \cos^2 x - \sin x \cos x$

$\Rightarrow \cos^2 x = -\sin x \cos x \rightarrow \cos x = -\sin x \rightarrow 2k\pi \cancel{\pm} x = x \Rightarrow 2x = -2k\pi$

$\Rightarrow x = -k\pi$

$\rightarrow \cos x = 0 \rightarrow 2k\pi \cancel{\pm} + \frac{\pi}{2}$

۴) $1/75$

الف) $\left(\cos \frac{\pi}{4} \cos x - \sin \frac{\pi}{4} \sin x\right)\left(\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}\right) = \frac{1}{4}$

$\Rightarrow \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x = \frac{1}{4} \Rightarrow \cos^2 x - \sin^2 x = \frac{1}{2} \Rightarrow \cos 2x = \frac{1}{2}$

$2k\pi + 2x = \frac{\pi}{3} \Rightarrow \frac{\pi}{3} - 2k\pi = 2x \Rightarrow \frac{\pi}{6} - k\pi = x$

$2k\pi - 2x = \frac{\pi}{3} \Rightarrow 2x = 2k\pi - \frac{\pi}{3} \Rightarrow x = k\pi - \frac{\pi}{6}$

$2x = k\pi \pm \frac{\pi}{3}$

$x = k\pi \pm \frac{\pi}{6}$

ب) $\cancel{\cos 2x} \Rightarrow 2k\pi + 2x - \frac{\pi}{9} = -(2k\pi + 2x) \Rightarrow 2k\pi + 2x - \frac{\pi}{9} = -2k\pi - 2x$

$-\sin 2x = \sin -2x$

$\Rightarrow 4x = -4k\pi + \frac{\pi}{9} \Rightarrow x = -k\pi + \frac{\pi}{36}$

$2k\pi - 2x + \frac{\pi}{9} = -(2k\pi + \pi \cancel{\pm} - 2x) \Rightarrow 2k\pi + \frac{\pi}{9} - 2x = -2k\pi - \pi + 2x$

$\Rightarrow 4x = -4k\pi + \frac{10\pi}{9} \Rightarrow x = -k\pi + \frac{5\pi}{18}$

$$\sin 4x = 2\sin 2x\cos 2x \Rightarrow \frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = 2\cos 2x + 1$$ ۱۰ (ه

$$\Rightarrow 2\cos 2x + \cancel{1} = \cos^2 x - \cancel{1}\sin^2 x \Rightarrow 2\cos 2x = 0 \Rightarrow 2\cos 2x = \cos 2\pi$$

$$\Rightarrow 2(2k\pi + 2x) = 2\pi \Rightarrow 4k\pi + 4x = 2\pi \Rightarrow 4x = 2\pi - 4k\pi \Rightarrow x = \pi/2 - k\pi$$

$$\Rightarrow 2(2k\pi - 2x) = 2\pi \Rightarrow 4k\pi - 4x = 2\pi \Rightarrow 4x = 4k\pi - 2\pi \Rightarrow x = k\pi - \pi/2$$

سوال ۵

$$\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \overbrace{\cancel{\sin^2 x + \cos^2 x}}^{1}$$

$$\frac{\cancel{\sin 2x}(2\cos 2x + 1)}{\cancel{\sin 2x}} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \rightarrow \cos 2x = 0$$

$$2x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = k\pi + \frac{\pi}{4}}$$

(۴ ۱,۲۵

$$\frac{\sin x}{\cos x+1} = \tan \frac{x}{2} \Rightarrow \frac{\cos t+1}{\sin t} = \frac{\sin t}{\cos t}$$

$$\frac{x}{2} = t$$

$$\cos^2 t + \cos t = \sin^2 t \Rightarrow \cos^2 t + \cos t = 1 - \cos^2 t \Rightarrow 2\cos^2 t + \cos t - 1 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1+8}}{4} \Rightarrow \cos t = -1 \;\Big|\; \cos t = \frac{1}{2}$$

$$\cos \frac{x}{2} = -1 \qquad \cos \pi = -1 \;\cancel{\Rightarrow \cos \frac{2\pi}{2} = -1}$$

$$\cos \frac{x}{2} = \frac{1}{2} \qquad \cos \frac{\pi}{3} = \frac{1}{2} \;\Big|\; \cos -\frac{\pi}{3} = \frac{1}{2} \Rightarrow x = \frac{2\pi}{3} \;\Big|\; x = -\frac{2\pi}{3} \Rightarrow$$ جواب‌ها تفاضل

$$\Rightarrow \left|\frac{2\pi}{3} - \left(-\frac{2\pi}{3}\right)\right| = \frac{4\pi}{3}$$

سوال ۴

$$\frac{\sin \frac{x}{2}}{1+\cos \frac{x}{2}} = \tan \frac{x}{4}$$

$$\frac{1}{\sin \frac{x}{2}} + \cot \frac{x}{2} = \frac{1+\cos \frac{x}{2}}{\sin \frac{x}{2}} = \frac{1}{\tan \frac{x}{4}} = \cot \frac{x}{4}$$

$$\tan \frac{x}{4} = \cot \frac{x}{2} \rightarrow \tan \frac{x}{4} = \tan\left(\frac{\pi}{2} - \frac{x}{2}\right)$$

$$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \pi$$

جواب‌ها $\rightarrow k = 0 \rightarrow x = \pi$

$\rightarrow k = -1 \rightarrow x = -\pi \longrightarrow$ اختلاف $= |\pi - (-\pi)| = 2\pi$

$1-\cos^2 x = -\sin^2 x \cos 3x \Rightarrow \cos 3x = -1 \to 2k\pi + 3x = +\pi \Rightarrow x = +\frac{\pi - 2k\pi}{3}$ (V

$\sin^2 x$ $\qquad \sin^2 x = 0 \qquad \to 2k\pi - 3x = \pi \Rightarrow x = \frac{2k\pi - \pi}{3}$

$\to k\pi$

$\to x = \{+\frac{\pi}{3}, \pi, \frac{5\pi}{3}, 0, 2k\pi\} \to$ ۵ جواب

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سوال ۹

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$

$1+\sin 2x = 2 - 2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0 \to \sin 2x = -1$

$\to \sin 2x = +\frac{1}{2}$

$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4}$

$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12}$ یا $\frac{5\pi}{12}$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، چون جواب رادیکال است نمی‌تواند

★ معادله ۲ جواب دارد

$$\frac{\tan x - 1}{\tan x + 1} = \tan\left(\frac{\pi}{4} - x\right) = \tan 3x \Rightarrow k\pi + 3x = \frac{\pi}{4} - x$$ (۸

$\Rightarrow$ ۲

$$\Rightarrow 4x = \frac{\pi}{4} - k\pi \Rightarrow x = \frac{\pi}{16} - \frac{k\pi}{4}$$

$$\sqrt{1 + 2\sin x \cos x} = \sqrt{2}\left(\cos^2 x - \sin^2 x\right) \Rightarrow |\cos x + \sin x| = \sqrt{2}\cos 2x$$ (۹

$\sin^2 x + \cos^2 x$

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$$\Rightarrow \sqrt{2}\left(\cos x + \sin x\right) = 1$$

$$\sqrt{2}\left(\cos x + \sin x\right) = -1$$

الف) $(\sin^2 x - \cos^2 x) \cdot 1 = 1 \Rightarrow \sin^2 x - \cos^2 x = 1 = \sin^2 x + \cos^2 x$ (۱۰

۲

$$\Rightarrow 2\cos^2 x = 0 \Rightarrow \cos^2 x = 0 \quad 2k\pi + \frac{\pi}{2} \longrightarrow \frac{\pi}{2}, \frac{3\pi}{2}$$

$$-\cos 2x = 1 \Rightarrow \cos 2x = -1 \Rightarrow 2k\pi + \cancel{2x} = \pi$$

ب) $\sin^3 x - \cos^3 x = -1$ $\longrightarrow x = 0 \longrightarrow -1$ ✓ $\Rightarrow x = \{0, \frac{3\pi}{2}, 2\pi\}$

$\longrightarrow x = \frac{\pi}{2} \longrightarrow \times$

$\longrightarrow x = \pi \longrightarrow 0 + 1 \times$

$\longrightarrow \frac{3\pi}{2} \longrightarrow$ ✓

PAPCO