نرگس زنده‌دل

۱)

الف) $\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow \cos 2x = 1 - 2\sin^2 x$

$\Rightarrow \underbrace{1 - 2\sin^2 x - 3\sin x + 1}_{} = 0 \Rightarrow \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{+3 \pm \sqrt{9+16}}{-4} = \frac{3 \pm 5}{-4} \begin{cases} \rightarrow -2 \\ \rightarrow 1/2 \ ✓ \end{cases}$

$\sin x = 1/2 \Rightarrow 2k\pi + \pi/6 \mid 2k\pi + \pi - \pi/6 = 2k\pi + 5\pi/6 \quad x = \{2k\pi + \pi/6 \ , \ 2k\pi + 5\pi/6\}$

ب) $\cos^2 x = \frac{1+\cos 2x}{2} \Rightarrow 2\cos^2 x - 1 = \cos 2x \Rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1+24}}{4} = \frac{-1 \pm 5}{4} \begin{cases} \rightarrow -3/2 \\ \rightarrow 1 \end{cases} \Rightarrow \cos x = 1 \Rightarrow x = \{2k\pi\}$

الف) $\sin^4 x - \cos^4 x = 1\cdot(\sin^2 x - \cos^2 x) = 2\sin 2x \cos 2x$ (۲

$\Rightarrow \sin^2 x - \cancel{\cos 2x} = 4\sin x \cos x(\cos^2 x - \sin^2 x) \Rightarrow \cos x = -\frac{1}{4}\sin x$

$\Rightarrow x = -\frac{1}{4}(2k\pi + x) \Rightarrow x = -\frac{1}{4}x - \frac{k\pi}{2} \Rightarrow \frac{5}{4}x = -\frac{k\pi}{2} \Rightarrow x = -\frac{2k\pi}{5}$

$x = -\frac{1}{4}(2k\pi + \pi - x) \Rightarrow x = -\frac{k\pi}{2} - \frac{\pi}{4} + \frac{1}{4}x \Rightarrow \frac{3}{4}x = -\frac{k\pi}{2} - \frac{\pi}{4} \Rightarrow x = -\frac{2k\pi}{3} - \frac{\pi}{3}$

ب) $2\cos^2 x - 1 = -2\sin x \cos x \Rightarrow \cos^2 x - \sin^2 x = -2\sin x \cos x \Rightarrow \cos 2x = -\sin 2x$

$-2x = 2k\pi + 2x \Rightarrow -4x = 2k\pi \Rightarrow x = -\frac{k\pi}{2}$

الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \rightarrow \sin x \neq 0 \Rightarrow \cos x(2\cos x + 1) = 1$ (۳

$\Rightarrow 2\cos^2 x + \cos x - 1 = 0 \rightarrow \cos x = -1 \rightarrow 2k\pi + \pi$

$1 \rightarrow -\frac{c}{a} = +\frac{1}{2} \rightarrow 2k\pi \pm \frac{\pi}{2}$

۳) ب) $2 - 2\sin^2 x = 2\cos^2 x \Rightarrow 2\cos^2 x = \cos^2 x - \sin x \cos x$

$\Rightarrow \cos^2 x = -\sin x \cos x \rightarrow \cos x = -\sin x \rightarrow 2k\pi + x = -x \Rightarrow 2x = -2k\pi$

$\Rightarrow x = -k\pi$

$\rightarrow \cos x = 0 \rightarrow 2k\pi$

۴) الف) $\left(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right)\left(\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}\right) = \frac{1}{4}$

$\Rightarrow \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x = \frac{1}{4} \Rightarrow \cos^2 x - \sin^2 x = \frac{1}{2} \Rightarrow \cos 2x = \frac{1}{2}$

$2k\pi + 2x = \frac{\pi}{3} \Rightarrow \frac{\pi}{3} - 2k\pi = 2x \Rightarrow \frac{\pi}{6} - k\pi = x$

$2k\pi - 2x = \frac{\pi}{3} \Rightarrow 2x = 2k\pi - \frac{\pi}{3} \Rightarrow x = k\pi - \frac{\pi}{6}$

ب) $\cancel{\cos 2x} \Rightarrow 2k\pi + 2x - \frac{\pi}{9} = -(2k\pi + 2x) \Rightarrow 2k\pi + 2x - \frac{\pi}{9} = -2k\pi - 2x$

$-\sin 2x = \sin -2x$

$\Rightarrow 4x = -4k\pi + \frac{\pi}{9} \Rightarrow x = -k\pi + \frac{\pi}{36}$

$2k\pi - 2x + \frac{\pi}{9} = -(2k\pi + \pi - 2x) \Rightarrow 2k\pi + \frac{\pi}{9} - 2x = -2k\pi - \pi + 2x$

$\Rightarrow 4x = -4k\pi + \frac{10\pi}{9} \Rightarrow x = -k\pi + \frac{5\pi}{18}$

(5

$\sin 4x = 2\sin 2x \cos 2x \Rightarrow \dfrac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = 2\cos 2x + 1$

$\Rightarrow 2\cos 2x + \cancel{1} = \cos^2 x \cancel{+} \sin^2 x \Rightarrow 2\cos 2x = 0 \Rightarrow 2\cos 2x = \cos 2\pi$

$\Rightarrow 2(2k\pi + 2x) = 2\pi \Rightarrow 4k\pi + 4x = 2\pi \Rightarrow 4x = 2\pi - 4k\pi \Rightarrow x = \pi/2 - k\pi$

$\Rightarrow 2(2k\pi - 2x) = 2\pi \Rightarrow 4k\pi - 4x = 2\pi \Rightarrow 4x = 4k\pi - 2\pi \Rightarrow x = k\pi - \pi/2$

$$\frac{\sin x}{\cos x+1}=\tan\frac{x}{2} \Rightarrow \frac{\cos t+1}{\sin t}=\frac{\sin t}{\cos t} \qquad (۴$$

$$\frac{x}{2}=t$$

$$\cos^2 t+\cos t=\sin^2 t \Rightarrow \cos^2 t+\cos t=1-\cos^2 t \Rightarrow 2\cos^2 t+\cos t-1=0$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1+8}}{4} \Rightarrow \cos t=-1 \;\big|\; \cos t=\frac{1}{2}$$

$$\cos\frac{x}{2}=-1 \qquad \cos\pi=-1 \Rightarrow \cos\frac{2\pi}{2}=-1$$

$$\cos\frac{x}{2}=\frac{1}{2} \qquad \cos\frac{\pi}{3}=\frac{1}{2} \;\Big|\; \cos-\frac{\pi}{3}=\frac{1}{2} \Rightarrow x=\frac{2\pi}{3} \;\Big|\; x=-\frac{2\pi}{3}$$ ⇒ ۲ جواب

$$\Rightarrow \left|\frac{2\pi}{3}-\left(-\frac{2\pi}{3}\right)\right|=\frac{4\pi}{3}$$

$1-\cos^2 x = -\sin^2 x \cos 3x \Rightarrow \cos 3x = -1 \rightarrow 2k\pi + 3x = +\pi \Rightarrow x = \frac{+\pi - 2k\pi}{3}$ (7

$\sin^2 x$

$\rightarrow 2k\pi - 3x = \pi \Rightarrow x = \frac{2k\pi - \pi}{3}$

$\sin^2 x = 0 \rightarrow k\pi + \cancel{}$

$\rightarrow x = \{+\frac{\pi}{3}, \pi, \frac{5\pi}{3}, \cancel{}, 0, 2k\pi, \cancel{}\} \rightarrow$ [illegible]

$\frac{\tan x - 1}{\tan x + 1} = \tan(\frac{\pi}{4} - x) = \tan 3x \Rightarrow k\pi + 3x = \frac{\pi}{4} - x$ (۸

$\Rightarrow 4x = \frac{\pi}{4} - k\pi \Rightarrow x = \frac{\pi}{16} - \frac{k\pi}{4}$

$\sqrt{\underbrace{1}_{\sin^2 x + \cos^2 x} + 2\sin x \cos x} = \sqrt{2}(\cos^2 x - \sin^2 x) \Rightarrow |\cos x + \sin x| = \sqrt{2}\cos 2x$ (۹

$\Rightarrow \sqrt{2}(\cos x + \sin x) = 1$

$\sqrt{2}(\cos x + \sin x) = -1$

الف) $(\sin^2 x - \cos^2 x)\, a\, 1 = 1 \Rightarrow \sin^2 x - \cos^2 x = 1 = \sin^2 x + \cos^2 x$ (۱۰

$\Rightarrow 2\cos^2 x = 0 \Rightarrow \cos^2 x = 0 \quad 2k\pi + \frac{\pi}{2} \rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

$-\cos 2x = 1 \Rightarrow \cos 2x = -1 \Rightarrow 2k\pi + 2x = \pi$

ب) $\sin^3 x - \cos^3 x = -1$
$\rightarrow x = 0 \rightarrow -1$ ✓
$\rightarrow x = \frac{\pi}{2} \rightarrow$ ✗
$\rightarrow x = \pi \rightarrow 0 + 1$ ✗
$\rightarrow \frac{3\pi}{2} \rightarrow$ ✓

$\Rightarrow x = \{0, \frac{3\pi}{2}, 2\pi\}$