نام و نام خانوادگی ....فاطمه........ پاسخنامه تشریحی تکلیف شماره ..۱۶.. کلاس ..دوازدهم تجربی A......

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الف) $\cos 2x = 3\sin x - 1$

$\frac{1-\cos 2x}{2} = \sin^2 x \Rightarrow \cos 2x = 1 - 2\sin^2 x$

$1 - 2\sin^2 x = 3\sin x - 1$

$2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \frac{-3 \pm \sqrt{9+16}}{4}$ → -2 غ ق ق ، → $\frac{1}{2} \Rightarrow \sin x = \sin\frac{\pi}{6}$

$x = 2k\pi + \frac{\pi}{6}$

$x = 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0$

$\frac{1+\cos 2x}{2} = \cos^2 x \Rightarrow \cos 2x = 2\cos^2 x - 1$

$2\cos^2 x + \cos x - 3 = 0$

$\cos x = \frac{-1 \pm \sqrt{1+24}}{4}$ → $-\frac{6}{4} = -\frac{3}{2}$ غ ق ق ، → $1 \Rightarrow \cos x = 1$

$x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin 4x$

$\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x}\underbrace{(\sin^2 x + \cos^2 x)}_{1} = 2\sin 2x\cos 2x$

$2\sin 2x\cos 2x + \cos 2x = 0$

$\cos 2x(2\sin 2x + 1) = 0$

$\Rightarrow \cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2}$ → $x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\Rightarrow \sin 2x = -\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$

$2x = 2k\pi - \frac{\pi}{6}$ → $x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi + \pi - \left(-\frac{\pi}{6}\right)$ → $x = k\pi + \frac{7\pi}{12}$

ب) $2\cos^2 x + 2\sin x\cos x = 1$

$\underbrace{2\cos^2 x - 1}_{} + \underbrace{2\sin x\cos x}_{} = 0$

$\cos 2x + \sin 2x = 0$

$\cos 2x = -\sin 2x = \sin(-2x) = \cos\left(\frac{\pi}{2} - (-2x)\right)$

$\cos 2x = \cos\left(\frac{\pi}{2} + 2x\right) \to 2x = 2k\pi + \frac{\pi}{2} + 2x$ غ ق ق

$\to 2x = 2k\pi - \frac{\pi}{2} - 2x$

$\Rightarrow 4x = 2k\pi - \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

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الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x}$

$\frac{\cos x}{\sin x}(2\cos x + 1) - \frac{1}{\sin x} = 0$

$\frac{1}{\sin x}(2\cos^2 x + \cos x - 1) = 0$

$\sin x \neq 0 \Rightarrow x \neq k\pi$

$\cos x = -1 \Rightarrow x = 2k\pi + \pi$ غ ق ق

$\cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3}$

ب) $3\sin^2 x - \sin x\cos x + \cos^2 x = 2$

$3\sin^2 x - 2 - \sin x\cos x + \cos^2 x = 0$

$2(\sin^2 x - 1) = -2\cos^2 x$

$-\sin x\cos x - \cos^2 x = 0 \Rightarrow -\cos x(\sin x + 1) = 0$

$\cos x = 0 \Rightarrow x = k\pi + \frac{\pi}{2}$

$\sin x = -1 \Rightarrow x = 2k\pi + \frac{3\pi}{2}$

→ $x = k\pi + \frac{\pi}{2}$

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الف) $\cos\left(x + \frac{\pi}{4}\right)\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{4}$

$\cos\left(x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4} - x\right)$

$\Rightarrow x + \frac{\pi}{4} + \frac{\pi}{4} - x = \frac{\pi}{2} \Rightarrow \cos\left(\frac{\pi}{4} - x\right) = \sin\left(x + \frac{\pi}{4}\right)$

$\Rightarrow \cos\left(x + \frac{\pi}{4}\right)\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2}\sin\left(2x + \frac{\pi}{2}\right) = \frac{1}{4}$

$\cos 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \Rightarrow x = k\pi \pm \frac{\pi}{6}$

ب) $\cos\left(2x - \frac{\pi}{9}\right) = -\sin 2x$

$\cos\left(2x - \frac{\pi}{9}\right) = \sin(-2x) = \cos\left(\frac{\pi}{2} + 2x\right)$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \Rightarrow$ غ ق ق

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \Rightarrow 4x = 2k\pi - \frac{\pi}{2} + \frac{\pi}{9}$

$x = \frac{k\pi}{2} - \frac{7\pi}{72}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \Rightarrow \frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x = 1$

مخرج $\neq 0$ → $\sin 2x \neq 0$

$2x \neq k\pi$

$x \neq \frac{k\pi}{2}$ *

$\Rightarrow 2\sin 2x\cos 2x + \sin 2x = \sin 2x$

$2\sin 2x\cos 2x = 0$

$\sin 2x = 0$ * غ ق ق

$\cos 2x = 0 \Rightarrow 2x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

ق ق

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$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$$

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \tan\frac{x}{4} = \frac{1}{\tan\frac{x}{4}} \Rightarrow \tan^2\frac{x}{4} = 1 \Rightarrow \tan\frac{x}{4} = \pm 1$$

مخرج $\neq 0$: $\cos\frac{x}{2} \neq -1$, $\sin\frac{x}{2} \neq 0$

$\frac{x}{2} \neq 2k\pi + \pi$, $\frac{x}{2} \neq k\pi$ *

$$\Rightarrow \frac{x}{4} = \frac{k\pi}{2} + \frac{\pi}{4} \Rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \text{ ق ق } \Rightarrow x = 2k\pi + \pi$$

$-\pi \leq x \leq \frac{3\pi}{2}$ $\Rightarrow x = -\pi, \pi$

$\Rightarrow x = |\pi - (-\pi)| = 2\pi$

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$$\cos^2 x - \sin^2 x \cos 3x = 1 \Rightarrow \underbrace{\cos^2 x - 1}_{-\sin^2 x} - \sin^2 x \cos 3x = 0$$

$$-\sin^2 x(1+\cos 3x) = 0$$

$\Rightarrow \sin x = 0 \Rightarrow x = k\pi \Rightarrow x = 0, \pi, 2\pi$ — جواب تکراری!

$$\Rightarrow 1+\cos 3x = 0 \Rightarrow \cos 3x = -1 \Rightarrow 3x = 2k\pi + \pi \Rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3} \Rightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

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$$\frac{1-\tan x}{1+\tan x} = \tan 2x \Rightarrow \tan\left(\frac{\pi}{4} - x\right) = \tan 2x \Rightarrow 2x = k\pi + \frac{\pi}{4} - x$$

$$\frac{1-\tan x}{1+\tan x} = \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4} \times \tan x}$$

$3x = k\pi + \frac{\pi}{4}$

$x = \frac{k\pi}{3} + \frac{\pi}{12}$ ق ق

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$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \Rightarrow \sqrt{1+2\sin x\cos x} = \sqrt{2}(\cos^2 x - \sin^2 x) \Rightarrow |\cos x + \sin x| = \sqrt{2}(\cos^2 x - \sin^2 x)$$

$$\xrightarrow{|\cos x + \sin x| \neq 0} \sqrt{2}|\cos x - \sin x| = 1 \Rightarrow \sqrt{2}|\sin x - \cos x| = 1$$

$$\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right) \Rightarrow \sqrt{2}\left|\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)\right| = 1 \Rightarrow 2\left|\sin\left(x-\frac{\pi}{4}\right)\right| = 1$$

$$\left|\sin\left(x-\frac{\pi}{4}\right)\right| = \frac{1}{2} \Rightarrow \sin\left(x-\frac{\pi}{4}\right) = \pm\frac{1}{2}$$  $$\Rightarrow x - \frac{\pi}{4} = k\pi \pm \frac{\pi}{6} \Rightarrow x = k\pi \pm \frac{\pi}{6} - \frac{\pi}{4} \Rightarrow$$ 

$\rightarrow$ ۲ جواب دارد $\Leftarrow [0, \pi]$

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الف) $\sin^4 x - \cos^4 x = 1$

$$(\sin^2 x - \cos^2 x)(\underbrace{\sin^2 x + \cos^2 x}_{1}) = 1$$

$$\Rightarrow \sin^2 x - \cos^2 x = 1$$

$$-\cos 2x = 1 \Rightarrow \cos 2x = -1$$

$2x = 2k\pi + \pi$

$x = k\pi + \frac{\pi}{2}$ 

$\Rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ را چک می‌کنیم:

$x = 0 \Rightarrow 0 - 1 = -1$ ✓

$x = \frac{\pi}{2} \Rightarrow 1 - 0 = 1$ ✗

$x = \pi \Rightarrow 0 - (-1) = 1$ ✗

$x = \frac{3\pi}{2} \Rightarrow -1 - 0 = -1$ ✓

$x = 2\pi \Rightarrow 0 - 1 = -1$ ✓

$0, \frac{3\pi}{2}, 2\pi$