~ ارنیا حسینی تکلیف ۱۶ دوازدهم دفتر A

۹/۲۵ پاسخنامه را با دقت مطالعه کن!

۱ (۱/۲۵)

$2x = 3\sin x - 1 \longrightarrow 1 - 2\sin^2 x = 3\sin x - 1 \longrightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\sin^2 x + 3\sin x - 4 = 0 \longrightarrow (\sin x - 3)(\sin x - 1) \longrightarrow \sin x = \frac{1}{2} \longrightarrow \sin x = \sin\frac{\pi}{6} \to x = 2k\pi + \frac{\pi}{6}$ ✓

ق غ $\frac{3}{2}$ ق و $\frac{1}{2}$ $x = 2k\pi + \pi - \frac{\pi}{6}$ ✓

$\cos^2 x = \frac{1+\cos 2x}{2}$ ↳ $\cos 2x = 2\cos^2 x - 1$

ب) $\cos 2x + \cos x - 2 = 0 \longrightarrow -1 + 2\cos^2 x + \cos x - 2 = 0$ ✗ $\longrightarrow 2\cos^2 x - \cos x + 1 = 0$

$2\cos^2 x + \cos x - 3 = 0 \longrightarrow \cos x = +1$ یا $-\frac{3}{2}$

$\cos^2 x - \cos x + 2 = 0 \to \cos x$ ✗ $\longrightarrow \Delta = 1 - 4\times 2 < 0 \to$ جواب ندارد

$\cos x = 1 \to x = 2k\pi$

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۲ (۲)

الف) $\sin^4 x - \cos^4 x = \sin 4x \longrightarrow \sin^2 x - \cos^2 x = \longrightarrow -\cos 2x = 2\sin 2x \cos 2x$

$2\sin 2x\cos 2x + \cos 2x = 0 \longrightarrow \cos 2x(2\sin 2x + 1) = 0$ → $\cos 2x = \cos\frac{\pi}{2} \longrightarrow 2x = 2k\pi \pm \frac{\pi}{2}$

$\sin 2x = -\frac{1}{2}$ → $\sin 2x = \sin -\frac{\pi}{6}$ → $2x = 2k\pi - \frac{\pi}{6}$; $2x = 2k\pi + \pi + \frac{\pi}{6}$ ✓

ب) $2\cos^2 x + 2\sin x\cos x = 1 \longrightarrow 2\cos^2 x - 1 = -2\sin x\cos x \longrightarrow \cos 2x = -\sin 2x$

$2x = k\pi - \frac{\pi}{4} \to x = \frac{k\pi}{2} - \frac{\pi}{8}$ ✓

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۳ (۲)

الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \longrightarrow \sin(\frac{\pi}{4} - x)\cos(\frac{\pi}{4} - x) = \frac{1}{4}$

$\frac{1}{2}\sin(\frac{\pi}{2} - 2x) = \frac{1}{4} \longrightarrow \cos 2x = \frac{1}{2} \to \cos 2x = \cos\frac{\pi}{3} \to 2x = 2k\pi \pm \frac{\pi}{3}$ ✓

ب) $\cos(2x - \frac{\pi}{6}) = -\sin 3x \longrightarrow \cos(2x - \frac{\pi}{6}) = \cos(3x + \frac{\pi}{2})$

→ $2x - \frac{\pi}{6} = 2k\pi + 2x + \frac{\pi}{2}$ غ ق

→ $2x - \frac{\pi}{6} = 2k\pi - 3x - \frac{\pi}{2}$

→ $4x = 2k\pi + \frac{\pi}{6} - \frac{\pi}{2}$ ✓

جابجا شد

$\cot x(2\cos x + 1) = \frac{1}{\sin x}$

$\longrightarrow \frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x}$

۴ (۰/۵)

$2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \longrightarrow 1 + \sin^2 x - \sin\cos x = 2$

$\rightarrow$

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۵ (۱/۵)

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \longrightarrow \sin 4x + \sin 2x = \sin 2x$

$\sin^2 x + \cos^2 x = 1$ ↳ $\sin 4x = 0 \longrightarrow \sin 4x = \sin 2x$

$\longrightarrow 4x = 2k\pi + 2x \longrightarrow x = \frac{k\pi}{2} + \frac{1}{2}\pi$

$4x = 2k\pi + \pi - 2x \longrightarrow x = \frac{k\pi}{2} - \frac{\pi}{4}$

سوال ۳

$$\frac{2\sin 2x\cos 2x + \sin 4x}{\sin 4x} = \sin^2 x + \cos^2 x = 1$$

$$\frac{\sin 4x(2\cos 4x + 1)}{\sin 4x} = 1 \xrightarrow{\sin 4x \neq 0} 2\cos 4x + 1 = 1 \rightarrow \cos 4x = 0$$

$$4x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = \frac{k\pi}{4} + \frac{\pi}{8}}$$

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$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \rightarrow \tan\frac{x}{4} = \frac{2\cos\frac{x}{4}}{2\sin\frac{x}{4}\cos\frac{x}{4}} \rightarrow \tan\frac{x}{4} = \cot\frac{x}{4}$$

$$\frac{x}{4} = k\pi + \frac{\pi}{4} \rightarrow x = 4k\pi + \pi \rightarrow \boxed{-\pi, \pi}$$

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$$\cos^2 2x - \sin^2 2x \cos^2 4x = 1$$

$$(1-\cos^2 2x)(4\cos^2 4x - 2\cos 2x) \rightarrow 4\cos^2 4x - 2\cos 2x - 4\cos^2 2x + 2\cos^3 2x$$

سوال ۴ قسمت الف)

$\cos(-x) = \cos x$

$$\cos\left(\frac{\pi}{2}+x\right)\cos\left(x-\frac{\pi}{2}\right) = \frac{1}{4} = \cos\left(\frac{\pi}{2}+x\right)\cos^*\left(\frac{\pi}{2}-x\right) = \frac{1}{4}$$

$$\text{if } \alpha + \beta = \frac{\pi}{2} \rightarrow \cos\alpha\sin\beta \rightarrow \cos\left(\frac{\pi}{2}+x\right)\sin\left(\frac{\pi}{2}+x\right) = \frac{1}{4}$$

$$\frac{1}{2}\sin\left(\frac{\pi}{2}+2x\right) = \frac{1}{4} \rightarrow \cos 2x = \frac{1}{2} \rightarrow 2x = 2k\pi \pm \frac{\pi}{3}$$

$$\boxed{x = k\pi \pm \frac{\pi}{6}}$$

سوال ۴ قسمت ب

$$\cos\left(2x - \frac{\pi}{2}\right) = -\sin 2x$$

$$\cos\left(2x - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} + 2x\right)$$

$$2x - \frac{\pi}{2} = 2k\pi \pm \left(\frac{\pi}{2} + 2x\right)$$

صلت ⊕ → $2k\pi + \frac{\pi}{2} + \frac{\pi}{2} = 0$ ✗

صلت ⊖ → $2x - \frac{\pi}{2} = 2k\pi - \frac{\pi}{2} - 2x$

$$4x = 2k\pi - \frac{7\pi}{12} \rightarrow x = \frac{k\pi}{2} - \frac{7\pi}{48}$$

سوال ۹

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$

$1+\sin 2x = 2-2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0$ → $\sin 2x = -1$ یا $\sin 2x = +\frac{1}{2}$

$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0\le x\le \pi} x = \frac{3\pi}{4}$ ✓

$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0\le x\le \pi} x = \frac{\pi}{12}$ ✓ یا $\frac{5\pi}{12}$ ✗

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x=\frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب رادیکال است مثبت است

* معادله ۲ جواب دارد

سوال ۱۰

در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا -1 فقط کافی است نقاط اصلی

ب ۱ چک کنی!

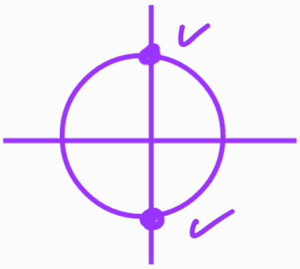
$\sin^3 x - \cos^3 x = 1$

$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

الف ۱

$\sin^4 x - \cos^4 x = 1$



$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

سوال ۷

$\cos^2 x - \sin^2 x \cos 4x = 1$ $\xrightarrow{\cos^2 x = 1 - \sin^2 x}$

$1 - \sin^2 x - \sin^2 x \cos 4x = 1 \longrightarrow \sin^2 x (1 + \cos 4x) = 0$ ← $\sin x = 0$ / ← $\cos 4x = -1$

$\sin x = 0 \longrightarrow x = k\pi \longrightarrow x = 0, \pi, 2\pi$ ③ ② ①

$\cos 4x = -1 \longrightarrow 4x = 2k\pi + \pi \longrightarrow x = \frac{2k\pi + \pi}{3} \longrightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ ⑤ ④

معادله ۵ جواب دارد

سوال ۸

$\tan \frac{\pi}{4} = 1 \longrightarrow \frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)$

$\tan\left(\frac{\pi}{4} - x\right) = \tan 4x \longrightarrow \frac{\pi}{4} - x = k\pi + 4x$

$-4x = k\pi - \frac{\pi}{4}$ $\xrightarrow{-k\pi = k\pi}$ $4x = k\pi + \frac{\pi}{4}$

$$x = \frac{k\pi}{4} + \frac{\pi}{16}$$