~ ارینا حسینی   تکلیف ۱۶   دوازدهم دفتر A

1

الف) $\cos 2x = 3\sin x - 1 \rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\sin^2 x + 3\sin x - 4 = 0 \rightarrow \underbrace{(\sin x - 2)}_{\frac{3}{2}\ \text{ق غ}}\underbrace{(\sin x - 1)}_{\frac{1}{2}\ \text{ق}} \rightarrow \sin x = \frac{1}{2} \rightarrow \sin x = \sin\frac{\pi}{6} \rightarrow x = 2k\pi + \frac{\pi}{6}$

$x = 2k\pi + \pi - \frac{\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 1 - 2\cos^2 x + \cos x - 2 = 0 \rightarrow 2\cos^2 x - \cos x + 1 = 0$

$\rightarrow \cos^2 x - \cos x + 2 = 0 \rightarrow \cos x\ X \rightarrow \Delta = 1 - 4\times 2 < 0 \rightarrow$ $\boxed{\text{جواب ندارد}}$

2

الف) $\sin^4 x - \cos^4 x = \sin 4x \rightarrow \sin^2 x - \cos^2 x = \rightarrow -\cos 2x = 2\sin 2x\cos 2x$

$2\sin 2x\cos 2x + \cos 2x = 0 \rightarrow \underbrace{\cos 2x}_{0}\underbrace{(2\sin 2x + 1)}_{\sin 2x = -\frac{1}{2}} = 0 \rightarrow \cos 2x = \cos\frac{\pi}{2} \rightarrow 2x = 2k\pi \pm \frac{\pi}{2}$

$\sin 2x = \sin -\frac{\pi}{6} \rightarrow 2x = 2k\pi - \frac{\pi}{6}$

$2x = 2k\pi + \pi + \frac{\pi}{6}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \rightarrow 2\cos^2 x - 1 = -2\sin x\cos x \rightarrow \cos 2x = -\sin 2x$

$2x = k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

3

الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \rightarrow \sin(\frac{\pi}{4} - x)\cos(\frac{\pi}{4} - x) = \frac{1}{4}$

$\frac{1}{2}\sin(\frac{\pi}{2} - 2x) = \frac{1}{4} \rightarrow \cos 2x = \frac{1}{2} \rightarrow \cos 2x = \cos\frac{\pi}{3} \rightarrow 2x = 2k\pi \pm \frac{\pi}{3}$

ب) $\cos(2x - \frac{\pi}{6}) = -\sin 2x \rightarrow \cos(2x - \frac{\pi}{6}) = \cos(2x + \frac{\pi}{2})$

$\rightarrow \cancel{2x} - \frac{\pi}{6} = 2k\pi + \cancel{2x} + \frac{\pi}{2}$ غ ق

$\rightarrow 2x - \frac{\pi}{6} = 2k\pi - 2x - \frac{\pi}{2}$

$\rightarrow \frac{4x}{4} = \frac{2k\pi + \frac{\pi}{6} - \frac{\pi}{2}}{4}$

جابجا شد

4

$\cot x(2\cos x + 1) = \frac{1}{\sin x}$

$\rightarrow \frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x}$

$2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \rightarrow 1 + \sin^2 x - \sin\cos x = 2$

$\rightarrow$

5

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \underbrace{\sin^2 x = \cos^2 x}_{\sin^2 x + \cos^2 x = 1} \rightarrow \sin 4x + \cancel{\sin 2x} = \cancel{\sin 2x}$

$\rightarrow \sin 4x = 0 \rightarrow \sin 4x = \sin 2x$

$\rightarrow 4x = 2k\pi + 2x \rightarrow x = \frac{k\pi}{2} + \frac{1}{2}\pi$

$4x = 2k\pi + \pi - 2x \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{4}$

$\boxed{۶}$

$$\frac{\sin\frac{x}{۲}}{1+\cos\frac{x}{۲}} = \frac{1}{\sin\frac{x}{۲}} + \frac{\cos\frac{x}{۲}}{\sin\frac{x}{۲}} \longrightarrow \tan\frac{x}{۴} = \frac{\cancel{۲}\cos\cancel{\frac{x}{۴}}}{\cancel{۲}\sin\frac{x}{۴}\cos\cancel{\frac{x}{۴}}} \longrightarrow \tan\frac{x}{۴} = \cot\frac{x}{۴}$$

$$\frac{x}{۴} = \frac{k\pi}{۲} + \frac{\pi}{۴} \longrightarrow x = ۲k\pi + \pi \longrightarrow \boxed{-\pi, \pi}$$

$\boxed{۷}$

$$\cos ۲x - \underbrace{\sin^۲ x}_{(1-\cos^۲ x)} \underbrace{\cos ۳x}_{(۴\cos^۳ x - ۳\cos x)} = 1 \longrightarrow$$

$$\longrightarrow ۴\cos^۳ x - ۳\cos x - ۴\cos^۵ x + ۳\cos^۳ x$$