حسابان ۲ - ۱۲ دفتر A

۱۸٫۵

۱) الف) $\cos 2x = 3\sin x - 1 \xrightarrow{\cos 2x = 1 - 2\sin^2 x} 1 - 2\sin^2 x - 3\sin x + 1 = 0$

$2\sin^2 x + 3\sin x - 2 = 0 \rightarrow \sin^2 x + 3\sin x - 4 = 0$ ← $\sin x = \frac{+4}{2} = 2$ × غیرقابل قبول

یا $\sin x = \frac{-1}{2}$ ✓

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$\sin x = -\frac{1}{2}$ $\quad x = 2k\pi - \frac{\pi}{6}$, $\quad x = 2k\pi + \pi + \frac{\pi}{6}$ $\rightarrow$ $x = 2k\pi - \frac{\pi}{6}$ ✓, $\quad x = 2k\pi + \frac{7\pi}{6}$ ✓

ب) $\cos 2x + \cos x - 2 = 0 \xrightarrow{\cos 2x = 2\cos^2 x - 1} 2\cos^2 x - 1 + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0 \rightarrow \cos x = 1$ ✓, $\quad \cos x = -\frac{3}{2}$ × غیرقابل قبول

$\cos x = 1 \quad x = 2k\pi$ ✓

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۲) الف) $\sin^4 x - \cos^4 x = \sin 4x \xrightarrow[\sin 4x = 2\sin 2x \cos 2x]{\cos 2x = \cos^4 x - \sin^4 x} -\cos 2x = 2\sin 2x \cos 2x$

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$\cos 2x(2\sin 2x + 1) = 0$ ← $\cos 2x = 0 \longrightarrow 2x = k\pi + \frac{\pi}{2}$ $\quad x = \frac{k\pi}{2} + \frac{\pi}{4}$ ✓

$2\sin 2x + 1 = 0 \rightarrow \sin 2x = -\frac{1}{2}$ یا $2x = 2k\pi - \frac{\pi}{6}$

$2x = 2k\pi + \pi + \frac{\pi}{6}$

ساده شدن $\longrightarrow$ $x = k\pi - \frac{\pi}{12}$, $\quad x = k\pi + \frac{7\pi}{12}$ ✓

ب) $2\cos^2 x + \underbrace{2\sin x \cos x}_{\sin 2x} = 1$ $\quad \underbrace{2\cos^2 x - 1}_{\cos 2x} + \sin 2x = 0$ $\quad \cos 2x = -\sin 2x$

$\cos 2x = \cos(\frac{\pi}{2} + 2x) \rightarrow 2x = 2k\pi + \frac{\pi}{2} + 2x \rightarrow$ ×

$2x = 2k\pi - \frac{\pi}{2} - 2x \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$ ✓

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۳) الف) $\frac{\cos x(2\cos x + 1)}{\cancel{\sin x}} = \frac{1}{\cancel{\sin x}}$ $\quad 2\cos^2 x + \cos x = 1$

$2\cos^2 x + \cos x - 1 = 0$ $\quad \cos x = -1 \rightarrow x = 2k\pi + \pi \xrightarrow{k=0} x = \pi \quad \sin \pi = 0$ × مخرج را صفر می‌کند

$\cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3}$ ✓

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$\rightarrow$ ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$ $\quad 2(\underbrace{\sin^2 x - 1}_{-\cos^2 x}) - \sin x\cos x + \cos^2 x = 0$

$-2\cos^2 x - \sin x\cos x + \cos^2 x = 0 \quad \cos^2 x + \sin x\cos x = 0 \quad \cos x(\cos x + \sin x) = 0$

$\rightarrow \cos x = 0 \quad x = 2k\pi$

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$\rightarrow \cos x + \sin x = 0 \quad \cos x = -\sin x \quad \rightarrow \cos x = \cos(\frac{\pi}{2} + x)$

$x = 2k\pi + \frac{\pi}{2} + x$ ✗

$x = 2k\pi - \frac{\pi}{2} - x \longrightarrow x = \frac{k\pi}{2} - \frac{\pi}{4}$ ✓

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الف) $\cos(\frac{\pi}{4} + x)\underbrace{\cos(x - \frac{\pi}{4})}_{= \cos(\frac{\pi}{4} - x)} = \frac{1}{4}$ $\quad \frac{\pi}{4} + x + \frac{\pi}{4} - x \xrightarrow[\text{زاویه}]{\text{جمع ۲}} \frac{\pi}{2} = 90^\circ$

وقتی ۲ زاویه متمم هستن $\rightarrow \sin x = \cos x$

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$\rightarrow \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4} \longrightarrow \frac{1}{2}\underbrace{\sin(\frac{\pi}{2} + 2x)}_{\cos 2x} = \frac{1}{4}$

$\rightarrow \cos 2x = \frac{1}{2} \quad 2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi \pm \frac{\pi}{6}$ ✓

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x$ $\quad$ ($\cos(\frac{\pi}{2} + 2x)$) $\quad \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x$ ✗

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \quad 4x = 2k\pi - \frac{9\pi}{18} + \frac{2\pi}{18} \quad x = \frac{k\pi}{2} - \frac{7\pi}{72}$

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$\dfrac{\sin 4x + \sin 2x}{\sin 2x} = \cancel{\cos^2 x + \sin^2 x}\ 1 \quad \sin 4x + \cancel{\sin 2x} = \cancel{\sin 2x}$

$\rightarrow \sin 2x = 0 \longrightarrow 2x = k\pi \quad x = \frac{k\pi}{2} \rightarrow x = \cancel{0}, \cancel{\frac{\pi}{2}}, \cancel{\pi}, \cancel{\frac{3\pi}{2}}$ $\quad$ ✗ $\leftarrow$ مخرج ۰ را صفر میکند

$\sin 4x = 0 \rightarrow 4x = k\pi, x = \frac{k\pi}{4} \rightarrow x = \cancel{0}, \frac{\pi}{4}, \cancel{\frac{\pi}{2}}, \frac{3\pi}{4}, \cancel{\pi}, \frac{5\pi}{4}$

$\cancel{\frac{3\pi}{2}}, \frac{7\pi}{4}$

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جوابها: $\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$ ✓

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$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \qquad \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$$

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$$\sin^2\frac{x}{2} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2} \qquad 1-\cos^2\frac{x}{2} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$$

$$2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \qquad 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right)=0$$

$\to 2\cos\frac{x}{2}=0 \qquad \frac{x}{2}=2k\pi \qquad x=4k\pi \longrightarrow$

k	-1	0	1
x	$\cancel{-4\pi}$	$\cancel{0}$	$\cancel{4\pi}$

$\to$ هیچ‌کدام در بازه نیست

$\to \cos\frac{x}{2}=-1 \qquad \frac{x}{2}=2k\pi+\pi \qquad x=4k\pi+2\pi \to$

k	-1	0	1
x	-2π	2π	6π

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$$\cos^2x-\sin^2x\cos^3x=1 \qquad -\sin^2x\cos^3x=\frac{1-\cos^2x}{\sin^2x}$$

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$$\sin^2x+\sin^2x\cos^3x=0 \qquad \sin^2x(1+\cos^3x)=0$$

$\sin^2x=0 \qquad \sin x=0 \qquad x=k\pi$

$\cos^3x=-1 \qquad 3x=2k\pi+\pi \qquad x=\frac{2k\pi}{3}+\frac{\pi}{3}$

$x=k\pi \longrightarrow$

k	0	1
x	2π	π

$\to \pi$ و 2π

$x=\frac{2k\pi}{3}+\frac{\pi}{3} \to$

k	0	1	2
x	$\frac{\pi}{3}$	π	$\frac{5\pi}{3}$

$\to \frac{\pi}{3}$ و π و $\frac{5\pi}{3}$

۵ جواب

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$$\frac{1-\tan x}{1+\tan x}=\tan^3x \qquad \frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}=\frac{\sin^3x}{\cos^3x} \qquad \frac{\frac{\cos x-\sin x}{\cancel{\cos x}}}{\frac{\cos x+\sin x}{\cancel{\cos x}}}=\frac{\sin^3x}{\cos^3x}$$

$$\frac{\cos x-\sin x}{\cos x+\sin x}=\frac{\sin^3x}{\cos^3x} \qquad \sin^3x\cos x+\sin x\sin^3x=\cos^3x\cos x-\cos^3x\sin x$$

$$\underbrace{\sin3x\cos x+\cos3x\sin x}_{\sin(3x+x)}=\underbrace{\cos3x\cos x-\sin x\sin3x}_{\cos(3x+x)}$$

2

$$\sin4x=\cos4x \qquad \sin4x=\sin\left(\frac{\pi}{2}-4x\right)$$

$4x=2k\pi+\frac{\pi}{2}-4x$

$4x=2k\pi+\pi-\frac{\pi}{2}+4x$ ✗

$$8x=2k\pi+\frac{\pi}{2} \qquad x=\frac{k\pi}{4}+\frac{\pi}{16}$$

⑨

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow[\otimes]{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

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$$\sin 2x = \underbrace{2\cos^2 2x - 1}_{\cos 4x} \qquad \cos 4x = \sin 2x \qquad 1 - 2\sin^2 2x = \sin 2x$$

$$2\sin^2 2x - \sin 2x - 1 = 0$$

$$\sin 2x = 1 \qquad 2x = 2k\pi + \frac{\pi}{2} \qquad x = k\pi + \frac{\pi}{4}$$

$$\sin 2x = -\frac{1}{2} \qquad 2x = 2k\pi - \frac{\pi}{6} \qquad x = k\pi - \frac{\pi}{12}$$

$$2x = 2k\pi + \frac{7\pi}{6} \qquad x = k\pi + \frac{7\pi}{12}$$

$$[0,\pi] \to x = \frac{2k\pi}{2} + \frac{\pi}{4} \qquad \begin{array}{c|cc} k & 0 & 1 \\ \hline x & \frac{\pi}{4} & \frac{5\pi}{4} \end{array} \to \frac{\pi}{4} \text{ و } \frac{5\pi}{4}$$

$$x = \frac{2k\pi}{2} - \frac{\pi}{12} \qquad \begin{array}{c|ccc} k & \cancel{0} & 1 & \cancel{2} \\ \hline x & \cancel{-\frac{\pi}{12}} & \frac{11\pi}{12} & \cancel{\frac{23\pi}{12}} \end{array} \qquad \frac{11\pi}{12}$$

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$$x = k\pi + \frac{7\pi}{12} \qquad \begin{array}{c|cc} k & 0 & \cancel{1} \\ \hline x & \frac{7\pi}{12} & \cancel{\frac{19\pi}{12}} \end{array} \qquad \frac{7\pi}{12}$$

⑩

الف) $\underbrace{\sin^4 x - \cos^4 x}_{-\cos 2x} = 1 \qquad \cos 2x = -\frac{1}{2}$

$$2x = 2k\pi + \frac{2\pi}{3}$$

$$2x = 2k\pi - \frac{2\pi}{3}$$

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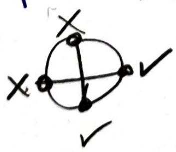
$$\to x = k\pi + \frac{\pi}{3} \to \begin{array}{c|cc} k & 0 & 1 \\ \hline x & \frac{\pi}{3} & \frac{4\pi}{3} \end{array} \qquad \frac{\pi}{3} \text{ و } \frac{4\pi}{3}$$

$$x = k\pi - \frac{\pi}{3} \to \begin{array}{c|ccc} k & \cancel{0} & 1 & 2 \\ \hline x & \cancel{-\frac{\pi}{3}} & \frac{2\pi}{3} & \frac{5\pi}{3} \end{array} \qquad \frac{2\pi}{3} \text{ , } \frac{5\pi}{3}$$

ب) $\sin^3 x - \cos^3 x = -1 \qquad (\sin x - \cos x)(\underbrace{\cancel{\sin^2 x + \cos^2 x}}_{1} + \sin x\cos x) = -1$

$$\sin x - \cos x = -1$$



$$x = 2\pi \text{ و } \frac{3\pi}{2}$$

$$1 + \frac{1}{2}\sin 2x = -1 \qquad -\frac{1}{2}\sin 2x = -2 \qquad \sin 2x = 4 \;\times$$

سوال ۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \tan\frac{x}{4}$$

$$\frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1}{\tan\frac{x}{4}} = \cot\frac{x}{4}$$

$$\tan\frac{x}{4} = \cot\frac{x}{4} \rightarrow \tan\frac{x}{4} = \tan\left(\frac{\pi}{2} - \frac{x}{4}\right)$$

$$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \pi$$

جواب ها $\quad k=0 \rightarrow x=\pi$

$\quad k=-1 \rightarrow x=-\pi \rightarrow$ اختلاف $= |\pi - (-\pi)| = 2\pi$

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2 - 2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0 \quad \begin{cases} \sin 2x = -1 \\ \sin 2x = +\frac{1}{2} \end{cases}$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12} \checkmark \text{ یا } \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب رادیکال است نمی‌تواند

* معادله ۲ جواب دارد

سوال ۱۰

در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا -1 فقط کافی است نقاط اصلی را چک کنی!

ب ۱

$$\sin^6 x - \cos^6 x = 1$$

$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

الف ۱

$$\sin^4 x - \cos^4 x = 1$$

$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$