۱)

الف) $\cos 2x = 3\sin x - 1 \xrightarrow{\cos 2x = 1-2\sin^2 x} 1 - 2\sin^2 x - 3\sin x + 1 = 0$

$2\sin^2 x + 3\sin x - 2 = 0 \rightarrow \sin^2 x + 3\sin x - 4 = 0$

$\sin x = \frac{+4}{2} = 2$ ✗ غیرقابل قبول

یا $\sin x = \frac{-1}{2}$ ✓

$\sin x = -\frac{1}{2}$ $\quad x = 2k\pi - \frac{\pi}{6}$, $\quad x = 2k\pi + \pi + \frac{\pi}{6}$ $\rightarrow$ $x = 2k\pi - \frac{\pi}{6}$, $x = 2k\pi + \frac{7\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \xrightarrow{\cos 2x = 2\cos^2 x - 1} 2\cos^2 x - 1 + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0 \rightarrow \cos x = 1$ ✓

$\cos x = -\frac{3}{2}$ ✗ غیرقابل قبول

$\cos x = 1$ $\quad x = 2k\pi$

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۲)

الف) $\sin^4 x - \cos^4 x = \sin 4x \xrightarrow[\sin 4x = 2\sin 2x\cos 2x]{\cos 2x = \cos^4 x - \sin^4 x} -\cos 2x = 2\sin 2x\cos 2x$

$\cos 2x(2\sin 2x + 1) = 0 \rightarrow \cos 2x = 0 \longrightarrow 2x = k\pi + \frac{\pi}{2}$ $\quad x = \frac{k\pi}{2} + \frac{\pi}{4}$

$2\sin 2x + 1 = 0 \rightarrow \sin 2x = -\frac{1}{2}$ یا $2x = 2k\pi - \frac{\pi}{6}$

$2x = 2k\pi + \pi + \frac{\pi}{6}$

$\xrightarrow{\text{بعد ساده شدن}}$ $x = k\pi - \frac{\pi}{12}$, $x = k\pi + \frac{7\pi}{12}$

ب) $2\cos^2 x + \underbrace{2\sin x\cos x}_{\sin 2x} = 1$ $\qquad \underbrace{2\cos^2 x - 1}_{\cos 2x} + \sin 2x = 0$ $\qquad \cos 2x = -\sin 2x$

$\cos 2x = \cos(\frac{\pi}{2} + 2x) \rightarrow 2x = 2k\pi + \frac{\pi}{2} + 2x \rightarrow$ ✗

$2x = 2k\pi - \frac{\pi}{2} - 2x \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

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۳)

الف) $\frac{\cos x(2\cos x + 1)}{\cancel{\sin x}} = \frac{1}{\cancel{\sin x}}$ $\qquad 2\cos^2 x + \cos x = 1$

$2\cos^2 x + \cos x - 1 = 0$

$\cos x = -1 \rightarrow x = 2k\pi + \pi \xrightarrow{k=0} x = \pi$ $\quad \sin\pi = 0$ ✗ مخرج را صفر می‌کند

$\cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3}$

$\rightarrow$ ۱) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$ $\quad 2\underbrace{(\sin^2 x - 1)}_{-\cos^2 x} - \sin x\cos x + \cos^2 x = 0$

$-2\cos^2 x - \sin x\cos x + \cos^2 x = 0$ $\quad \cos^2 x + \sin x\cos x = 0$ $\quad \cos x(\cos x + \sin x) = 0$

$\rightarrow \cos x = 0$ $\quad x = 2k\pi$

$\rightarrow \cos x + \sin x = 0$ $\quad \cos x = -\sin x$ $\rightarrow \cos x = \cos(\frac{\pi}{2} + x)$

$x = 2k\pi + \frac{\pi}{2} + x$ ✗

$x = 2k\pi - \frac{\pi}{2} - x \longrightarrow x = \frac{k\pi}{2} - \frac{\pi}{4}$

۴

الف) $\underbrace{\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4})}_{=\cos(\frac{\pi}{4} - x)} = \frac{1}{4}$ $\quad \frac{\pi}{4} + x + \frac{\pi}{4} - x \xrightarrow[\text{زاویه}]{\text{جمع ۲}} \frac{\pi}{2} = 90^\circ$

وقتی ۲ زاویه متمم هستن $\leftarrow \sin x = \cos x$

$\rightarrow \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4} \longrightarrow \frac{1}{2}\underbrace{\sin(\frac{\pi}{2} + 2x)}_{\cos 2x} = \frac{1}{4}$

$\rightarrow \cos 2x = \frac{1}{2}$ $\quad 2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x$ $\quad$ ($\cos(\frac{\pi}{2} + 2x)$) $\quad \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x$ ✗

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x$ $\quad 4x = 2k\pi - \frac{9\pi}{18} + \frac{2\pi}{18}$ $\quad x = \frac{k\pi}{2} - \frac{7\pi}{72}$

۵

$\frac{\sin 4x + \sin 2x}{\sin 2x} = \cancel{\cos^2 x + \sin^2 x}$ (۱) $\quad \sin 4x + \cancel{\sin 2x} = \cancel{\sin 2x}$

$\rightarrow \sin 2x = 0 \longrightarrow 2x = k\pi$ $\quad x = \frac{k\pi}{2} \rightarrow x = \cancel{0}, \cancel{\frac{\pi}{2}}, \cancel{\pi}, \cancel{\frac{3\pi}{2}}$ $\quad$ X ← مخرج صفر میشه

$\sin 4x = 0 \rightarrow 4x = k\pi, x = \frac{k\pi}{4} \rightarrow x = \cancel{0}, \frac{\pi}{4}, \cancel{\frac{\pi}{2}}, \frac{3\pi}{4}, \cancel{\pi}, \frac{5\pi}{4}, \cancel{\frac{3\pi}{2}}, \frac{7\pi}{4}$

جوابها: $\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

(6)

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$ $\quad \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$\sin^2\frac{x}{2} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$ $\quad 1-\cos^2\frac{x}{2} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$

$2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0$ $\quad 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right)=0$

$\to 2\cos\frac{x}{2}=0 \quad \frac{x}{2}=2k\pi \quad x=4k\pi \longrightarrow \begin{array}{c|ccc} k & -1 & 0 & 1 \\ \hline x & \cancel{-4\pi} & \cancel{0} & \cancel{4\pi} \end{array} \to$ هیچ جوابی در بازه نیست

$\to \cos\frac{x}{2}=-1 \quad \frac{x}{2}=2k\pi+\pi \quad x=4k\pi+2\pi \to \begin{array}{c|ccc} k & -1 & 0 & 1 \\ \hline x & -2\pi & 2\pi & 6\pi \end{array}$

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(7)

$\cos^2x-\sin^2x\cos^3x=1 \quad -\sin^2x\cos^3x=\underbrace{1-\cos^2x}_{\sin^2x}$

$\sin^2x+\sin^2x\cos^3x=0 \quad \sin^2x(1+\cos^3x)=0$

$\to \sin^2x=0 \quad \sin x=0 \quad x=k\pi$

$\to \cos^3x=-1 \quad 3x=2k\pi+\pi \quad x=\frac{2k\pi}{3}+\frac{\pi}{3}$

$x=k\pi \longrightarrow \begin{array}{c|cc} k & 0 & 1 \\ \hline x & 2\pi & \pi \end{array} \to \pi$ و 2π

$x=\frac{2k\pi}{3}+\frac{\pi}{3} \to \begin{array}{c|ccc} k & 0 & 1 & 2 \\ \hline x & \frac{\pi}{3} & \pi & \frac{5\pi}{3} \end{array} \to \frac{\pi}{3}$ و π و $\frac{5\pi}{3}$

۵ جواب

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(8)

$\frac{1-\tan x}{1+\tan x}=\tan^3x \quad \frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}=\frac{\sin^3x}{\cos^3x} \quad \frac{\frac{\cos x-\sin x}{\cancel{\cos x}}}{\frac{\cos x+\sin x}{\cancel{\cos x}}}=\frac{\sin^3x}{\cos^3x}$

$\frac{\cos x-\sin x}{\cos x+\sin x}=\frac{\sin^3x}{\cos^3x} \quad \sin3x\cos x+\sin x\sin3x=\cos3x\cos x-\cos3x\sin x$

$\underbrace{\sin3x\cos x+\cos3x\sin x}_{\sin(3x+x)}=\underbrace{\cos3x\cos x-\sin x\sin3x}_{\cos(3x+x)}$

$\sin4x=\cos4x \quad \sin4x=\sin\left(\frac{\pi}{2}-4x\right) \to 4x=2k\pi+\frac{\pi}{2}-4x$

$4x=2k\pi+\pi-\frac{\pi}{2}+4x \; ✗$

$8x=2k\pi+\frac{\pi}{2} \quad x=\frac{k\pi}{4}+\frac{\pi}{16}$

(۹)

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow[\otimes]{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$

$\sin 2x = \underbrace{2\cos^2 2x - 1}_{\cos 4x}$ $\quad \cos 4x = \sin 2x \quad 1 - 2\sin^2 2x = \sin 2x$

$2\sin^2 2x - \sin 2x - 1 = 0$

$\sin 2x = 1 \quad 2x = 2k\pi + \frac{\pi}{2} \quad x = k\pi + \frac{\pi}{4}$

$\sin 2x = -\frac{1}{2} \quad 2x = 2k\pi - \frac{\pi}{6} \quad x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi + \frac{7\pi}{6} \quad x = k\pi + \frac{7\pi}{12}$

$[0,\pi] \to x = \frac{2k\pi}{2} + \frac{\pi}{4}$ $\quad \begin{array}{c|cc} k & 0 & 1 \\ \hline x & \frac{\pi}{4} & \frac{5\pi}{4} \end{array} \to \left(\frac{\pi}{4} \text{ و } \frac{5\pi}{4}\right)$

$x = \frac{2k\pi}{2} - \frac{\pi}{12}$ $\quad \begin{array}{c|ccc} k & \cancel{0} & 1 & \cancel{2} \\ \hline x & \cancel{-\frac{\pi}{12}} & \frac{11\pi}{12} & \cancel{\frac{23\pi}{12}} \end{array}$ $\quad \frac{11\pi}{12}$

۴ جواب

$x = k\pi + \frac{7\pi}{12}$ $\quad \begin{array}{c|cc} k & 0 & \cancel{1} \\ \hline x & \frac{7\pi}{12} & \cancel{\frac{19\pi}{12}} \end{array}$ $\quad \frac{7\pi}{12}$

(۱۰)

الف) $\underbrace{\sin^4 x - \cos^4 x}_{-\cos 2x} = 1$ $\quad \cos 2x = -\frac{1}{2}$ $\quad 2x = 2k\pi + \frac{2\pi}{3}$

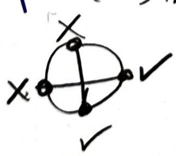
$2x = 2k\pi - \frac{2\pi}{3}$

$\to x = k\pi + \frac{\pi}{3} \to \begin{array}{c|cc} k & 0 & 1 \\ \hline x & \frac{\pi}{3} & \frac{4\pi}{3} \end{array}$ $\quad \frac{\pi}{3}$ و $\frac{4\pi}{3}$

$x = k\pi - \frac{\pi}{3} \to \begin{array}{c|ccc} k & \cancel{0} & 1 & 2 \\ \hline x & \cancel{-\frac{\pi}{3}} & \frac{2\pi}{3} & \frac{5\pi}{3} \end{array}$ $\quad \frac{2\pi}{3} , \frac{5\pi}{3}$

ب) $\sin^3 x - \cos^3 x = -1$ $\quad (\sin x - \cos x)(\underbrace{\sin^2 x + \cos^2 x}_{1} + \sin x\cos x) = -1$

$\sin x - \cos x = -1$



$x = 2\pi$ و $\frac{3\pi}{2}$

$1 + \frac{1}{2}\sin 2x = -1 \quad -\frac{1}{2}\sin 2x = -2 \quad \sin 2x = 4 \quad \times$