/ /　　　　　　۱۵/۵　　　　　　سحر کریمی

۱) ۱/۲۵

الف) $\cos 2x = 3\sin x - 1 \rightarrow 1 - \cos 2x = 3\sin x$

فرمول طلایی $2\sin^2 x = 3\sin x \rightarrow \begin{cases} \sin x = 0 \checkmark \\ \sin x = \frac{3}{2} \times \end{cases}$

$x = 2k\pi + \frac{\pi}{6}$

$x = 2k\pi + \frac{5\pi}{6}$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x + 3\sin x - 2 = 0$

$\boxed{x = k\pi}$

یا $\sin^2 x + 3\sin x - 2 = 0$ → $\sin x = \frac{1}{2} \checkmark$

یا $\sin x = -2 \times$

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0 \rightarrow \cos^2 x + \cos x - 6 = 0 \rightarrow$

$(\cos x + 3)(\cos x - 2) = 0 \rightarrow \begin{cases} \cos x = \frac{2}{2} = 1 \checkmark \\ \cos x = \frac{-3}{2} \times \end{cases}$

$\boxed{x = 2k\pi}$

۲) ۲

الف) $(\underbrace{\sin^2 + \cos^2}_{1})(\sin^2 - \cos^2) = \sin 4x \rightarrow -\cos 2\alpha = \sin 4\alpha$

$2\sin 2\alpha\, \cancel{\cos 2\alpha} = -\cancel{\cos 2\alpha} \rightarrow \sin 2\alpha = \frac{-1}{2} \rightarrow \begin{cases} 2\alpha = 2k\pi - \frac{\pi}{6} \\ 2\alpha = 2k\pi - \frac{5\pi}{6} \end{cases}$

$\rightarrow \begin{cases} \alpha = k\pi - \frac{\pi}{12} \\ \alpha = k\pi - \frac{5\pi}{12} \end{cases}$

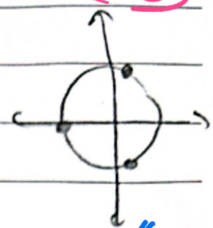
ب) $2\cos^2 x + \sin 2x = 1 \rightarrow 2\cos^2 x - 1 + \sin 2x = 0$

$\cos 2\alpha = -\sin 2\alpha \rightarrow \begin{cases} 2\alpha = 2k\pi + \frac{3\pi}{4} \rightarrow \alpha = k\pi + \frac{3\pi}{8} \\ 2\alpha = 2k\pi + \frac{7\pi}{4} \rightarrow \alpha = k\pi + \frac{7\pi}{8} \end{cases}$

۳) الف) $\frac{\cos x}{\sin x}(2\cos x+1)=\frac{1}{\sin x} \rightarrow \sin x \neq 0$

۱/۷۵

$2\cos^2\alpha+\cos\alpha-1=0 \rightarrow \begin{cases}\cos\alpha=-1\\ \cos\alpha=\frac{1}{2}\end{cases}$



ج: $\begin{cases}2k\pi+\pi \times \\ 2k\pi\pm\frac{\pi}{3}\end{cases}$

اگر $\cos x=-1$ باشد، $\sin x=0$ است و عبارت تعریف نشده میشود

پس جواب غیرقابل قبول محسوب میشود

ب) $\sin^2 x+1-\sin x\cos x=2 \rightarrow -\sin x\cos x=\cos^2\alpha$

$\begin{cases}\cos\alpha=0 \rightarrow \alpha=2k\pi\\ -\sin x=\cos\alpha \rightarrow \begin{cases}\alpha=2k\pi+\frac{3\pi}{4}\\ \alpha=2k\pi+\frac{7\pi}{4}\end{cases}\end{cases}$

۴) الف) $\xrightarrow[\text{خارج است}]{\text{Cos منفی}}$ $\cos(\frac{\pi}{4}+x)\cos(\frac{\pi}{4}-x)=\frac{1}{4}$

۲

$\rightarrow \frac{\pi}{4}+x+\frac{\pi}{4}-x=\frac{\pi}{2} \rightarrow \cos(\frac{\pi}{4}+x)\sin(\frac{\pi}{4}+x)=\frac{1}{4}$

$\sin(x+\frac{\pi}{2})=\frac{1}{2} \rightarrow \begin{cases}x+\frac{\pi}{2}=2k\pi+\frac{\pi}{6} \rightarrow \boxed{x=2k\pi-\frac{\pi}{3}}\\ x+\frac{\pi}{2}=2k\pi+\frac{5\pi}{6}\end{cases}$

$\boxed{x=2k\pi+\frac{\pi}{3}}$

/ /

ب) $\sin\left(\frac{\pi}{4} - 2x + \frac{\pi}{9}\right) = \sin(-2x)$

$$\begin{cases} \cancel{-2x} = 2k\pi + \frac{\pi}{4} \cancel{- 2x} + \frac{\pi}{4} \rightarrow 2k\pi = -\pi \rightarrow k \notin \mathbb{Z} \rightarrow \times \\ 2x = 2k\pi - \frac{\pi}{4} + 2x - \frac{\pi}{9} \rightarrow -4x = 2k\pi - \frac{11\pi}{18} \end{cases}$$

$$\boxed{x = -\frac{k\pi}{2} + \frac{11\pi}{72}}$$

۵) $\frac{\sin 4x + \sin 2x}{\sin 2x} = 1 \rightarrow \sin 4x + \cancel{\sin 2x} = \cancel{\sin 2x}$

۱/۵

$\sin 4x = 0 \rightarrow 4x = k\pi \rightarrow \boxed{x = \frac{k\pi}{4}}$

۶) $\frac{x}{2} = \alpha \rightarrow \frac{\sin\alpha}{1+\cos\alpha} = \underbrace{\frac{1}{\sin\alpha} + \cot\alpha}_{\frac{\cos\alpha + 1}{\sin\alpha}}$

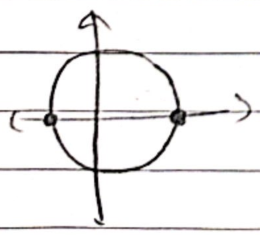
۲

$\rightarrow \tan\frac{\alpha}{2} = \cot\frac{\alpha}{2} \rightarrow \tan\frac{x}{4} = \cot\frac{x}{4} \rightarrow \left\{ \frac{x}{4} = k\pi + \frac{\pi}{4} \right.$

$\rightarrow \{ x = 2k\pi + \pi \rightarrow \begin{cases} k = -1 \rightarrow x = -\pi \\ k = 0 \rightarrow x = \pi \end{cases} \quad |\cancel{-2\pi}| = 2\pi$

۷) $\sin^2\alpha \times \cos^3\alpha = \sin^2\alpha \rightarrow \begin{cases} \sin^2\alpha = 0 \rightarrow \alpha = k\pi \\ \cos^3\alpha = -1 \rightarrow \alpha = 2k\pi + \pi \\ \cos\alpha = -1 \end{cases}$

۱/۲۵



$\rightarrow$ جواب ۳

۸)
$$\frac{1-\tan\alpha}{1+\tan\alpha} \xrightarrow{\times(1+\tan\alpha)} \frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \cos 2\alpha$$

۱۵

$$\cos 2\alpha = \frac{\sin 3x}{\cos 3x} \to \cos 2\alpha \cos 3\alpha = \sin 3\alpha \to$$

۹)
$$1+\sin x = 2\frac{\cos 2\alpha}{1-2\sin^2\alpha} \to 1+\sin\alpha = 2-4\sin^2\alpha$$

$$\to 4\sin^2\alpha + \sin\alpha - 1 = 0 \to \left\{ \sin x = \frac{\pm\sqrt{17}-1}{8} \right.$$

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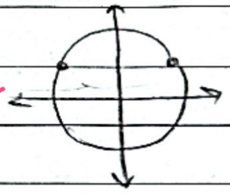
$$\xrightarrow{[0,\pi]} \sin\alpha = \frac{\sqrt{17}-1}{8}$$



→ ۲ جواب دارد

۱۰) الف

$$(\sin^2 - \cos^2)(\cancel{\sin^2+\cos^2}) = 1 \to \sin^2 - 1 = \cos^2\alpha$$

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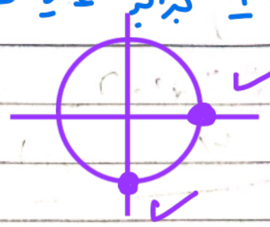
$$\to -\cos^2\alpha = \cos^2\alpha \to \cos\alpha = 0 \to \alpha = k\pi + \frac{\pi}{2}$$

$$\to \alpha = \frac{\pi}{2}, \frac{3\pi}{2}$$

ب) در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا 1- فقط کافی است نقاط اصلی را چک کنی!

$$\sin^4 x - \cos^4 x = 1$$



$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

سوال ۵

$$\frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = \overbrace{\sin^2 x + \cos^2 x}^{1}$$

$$\frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \rightarrow \cos 2x = 0$$

$$2x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = k\pi + \frac{\pi}{4}}$$

سوال ۸

$$\tan\frac{\pi}{4} = 1 \rightarrow \frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)$$

$$\tan\left(\frac{\pi}{4} - x\right) = \tan 3x \rightarrow \frac{\pi}{4} - x = k\pi + 3x$$

$$-4x = k\pi - \frac{\pi}{4} \xrightarrow{-k\pi = k\pi} 4x = k\pi + \frac{\pi}{4}$$

$$\boxed{x = \frac{k\pi}{4} + \frac{\pi}{16}}$$