Date No

آیلار منوچهرنیا

۲۰ آفرین خیلی عالی نوشتی!

۱. الف) $\cos 2x = 3\sin x - 1$

$\cos^2 x - \sin^2 x = 3\sin x - 1$

$1 - \sin^2 x - \sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$ ۲

$\sin x = \frac{-3 \pm \sqrt{9+16}}{4}$ → $\frac{1}{2}$ → $x = 2k\pi + \frac{\pi}{6}$ / $2k\pi + \frac{5\pi}{6}$ ✓ ✓

→ -2 غ ق ق

ب) $\cos 2x + \cos x - 2 = 0$

$\cos^2 x - \sin^2 x$

$\cos^2 x - 1 + \cos^2 x + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\cos x$ → 1 → $x = 2k\pi$ ✓

→ $\frac{c}{a} = \frac{-3}{2}$ غ ق ق

۲. الف) $\sin^4 x - \cos^4 x = \sin 4x$

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$ ۲

1 $-\cos 2x = 2\sin 2x \cos 2x$

$2\sin 2x \cos 2x + \cos 2x = 0 \rightarrow \cos 2x(2\sin 2x + 1) = 0$

→ $\cos 2x = 0 \rightarrow 2x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

→ $\sin 2x = \frac{-1}{2} \rightarrow 2x = 2k\pi - \frac{\pi}{6} \rightarrow k\pi - \frac{\pi}{12}$

→ $2k\pi + \frac{7\pi}{6} \rightarrow k\pi + \frac{7\pi}{12}$ ✓ ✓

آیلا فیروزی‌نیا

ب) $2\cos^2 x + 2\sin x \cos x = 1$

$1 + \cos 2x + \sin 2x = 1 \rightarrow \cos 2x + \sin 2x = 0$

$\cos 2x = -\sin 2x \longrightarrow \sin(-2x) = \cos(\frac{\pi}{2} - (-2x))$

$\cos 2x = \cos(\frac{\pi}{2} + 2x) \longrightarrow 2x = 2k\pi + \frac{\pi}{2} + 2x$ غ ق ق

$\longrightarrow 2x = 2k\pi - \frac{\pi}{2} - 2x$

$4x = 2k\pi - \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

۳- الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x}$

$\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \longrightarrow \frac{1}{\sin x}(2\cos^2 x + \cos x - 1) = 0$

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$\cos x \longrightarrow -1 \longrightarrow 2k\pi + \pi$ غ ق ق $(\sin = 0)$

$\longrightarrow \frac{-c}{a} = \frac{1}{2} \longrightarrow x = 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x \cos x + \cos^2 x = 2$

$2(\sin^2 x - 1) = -2\cos^2 x \longrightarrow -\sin x \cos x - \cos x = 0 \longrightarrow -\cos x(\sin x + 1) = 0$

$\cos x = 0 \longrightarrow x = k\pi + \frac{\pi}{2}$

$\sin x = -1 \longrightarrow x = 2k\pi - \frac{\pi}{2}$ } $k\pi + \frac{\pi}{2}$

۴- الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4}$

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$\cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} - x)$

$x + \frac{\pi}{4} + \frac{\pi}{4} - x = \frac{\pi}{2} \longrightarrow \cos(\frac{\pi}{4} - x) = \sin(x + \frac{\pi}{4})$

$\longrightarrow \cos(x + \frac{\pi}{4})\sin(x + \frac{\pi}{4}) = \frac{1}{2}\sin(2x + \frac{\pi}{2}) = \frac{1}{4}$

$\cos 2x = \frac{1}{2} \longrightarrow 2x = 2k\pi \pm \frac{\pi}{3} \longrightarrow x = k\pi \pm \frac{\pi}{6}$

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آیلا فیروزنیا

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x$

$\cos(2x - \frac{\pi}{9}) = \sin(-2x) = \cos(\frac{\pi}{2} + 2x) \to 2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \to$ غ ق ق

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \to 4x = 2k\pi - \frac{7\pi}{18} \to x = \frac{k\pi}{2} - \frac{7\pi}{72}$

۵- $\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x - \cos^2 x \to \frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x}$ (2)

$\sin 2x \neq 0 \to 2x \neq k\pi \to x \neq \frac{k\pi}{2}$

$= \sin^2 x + \cos^2 x = 1$

$\to 2\sin 2x \cos 2x + \sin 2x = \sin 2x$

$\to \sin 2x = 0 \to 2x = k\pi \to x = \frac{k\pi}{2}$ غ ق ق

$\cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{2} + \frac{\pi}{4}$

۶- $\frac{\sin\frac{x}{2}}{1 + \cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$= \frac{1 + \cos\frac{x}{2}}{\sin\frac{x}{2}}$ $\quad 1 + \cos\frac{x}{2} \neq 0 \to \cos\frac{x}{2} \neq -1 \to \frac{x}{2} \neq 2k\pi + \pi$ (2)

$\sin\frac{x}{2} \neq 0 \to \frac{x}{2} \neq k\pi$

$\tan\frac{x}{4} = \frac{1}{\tan\frac{x}{2}} \to \tan^2\frac{x}{4} = 1 \to \tan\frac{x}{4} = \pm 1$

$\to \frac{x}{4} = \frac{k\pi}{2} + \frac{\pi}{4} \to \frac{x}{2} = k\pi + \frac{\pi}{2} \to x = 2k\pi + \pi$

$-\pi \leq x \leq \frac{3\pi}{2} \to x = -\pi, \pi$

$x = |\pi - (-\pi)| = 2\pi$

آیلا فرجادنیا

۷ـ $\cos^2 x - \sin^2 x \cos^3 x = 1 \rightarrow \underbrace{\cos^2 x - 1}_{-\sin^2 x} - \sin^2 x \cos^3 x = 0$

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$-\sin^2 x (1 + \cos^3 x) = 0$

$\rightarrow \sin x = 0 \rightarrow x = k\pi \rightarrow x = 0, \pi, 2\pi$

$\rightarrow 1 + \cos 3x = 0 \rightarrow \cos 3x = -1 \rightarrow 3x = 2k\pi + \pi \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$

$\rightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

(۵ جواب) جواب‌ها $= 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

۸ـ $\frac{1 - \tan x}{1 + \tan x} = \tan 3x \rightarrow \tan\left(\frac{\pi}{4} - x\right) = \tan 3x \rightarrow 3x = k\pi + \frac{\pi}{4} - x$

$4x = k\pi + \frac{\pi}{4}$

$\frac{1 - \tan x}{1 + \tan x} = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}$

$\rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$

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۹ـ $\sqrt{1 + \sin 2x} = \sqrt{2}\cos 2x \rightarrow \sqrt{1 + 2\sin x \cos x} = \sqrt{2}(\cos^2 x - \sin^2 x)$

$\rightarrow |\cos x + \sin x| = \sqrt{2}(\cos^2 x - \sin^2 x) \xrightarrow{|\cos x + \sin x| \neq 0} \sqrt{2}\,|\cos x - \sin x| = 1$

$\rightarrow \sqrt{2}\,|\sin x - \cos x| = 1$

$\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right) \rightarrow \sqrt{2}\left|\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)\right| = 1 \rightarrow \sqrt{2}\cdot\sqrt{2}\left|\sin\left(x - \frac{\pi}{4}\right)\right| = 1$

$\left|\sin\left(x - \frac{\pi}{4}\right)\right| = \frac{1}{2} \rightarrow \sin\left(x - \frac{\pi}{4}\right) = \pm\frac{1}{2} \rightarrow x - \frac{\pi}{4} = k\pi \pm \frac{\pi}{6}$

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$x = k\pi + \frac{5\pi}{12}$

$x = k\pi + \frac{\pi}{12}$

۲ جواب دارد در بازه‌ی $[0, \pi]$

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آیلا فیروزنیا

۱۰-

الف) $\sin^4 x - \cos^4 x = 1$

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$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

1

$\sin^2 x - \cos^2 x = 1$

$-\cos 2x = 1 \rightarrow \cos 2x = -1 \rightarrow 2x = 2k\pi + \pi$

$x = k\pi + \frac{\pi}{2} \rightarrow \frac{\pi}{2} / \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$

$\sin x = 0 / \cos x = +1 \rightarrow 0, 2\pi$

$\sin x = -1 / \cos x = 0 \rightarrow \frac{3\pi}{2}$