Date No

تمرین

۱- الف) $\cos 2x = 3\sin x - 1$

$\cos^2 x - \sin^2 x = 3\sin x - 1$

$1 - \sin^2 x - \sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$ → $\frac{1}{2}$ → $x = 2k\pi + \frac{\pi}{6}$ / $2k\pi + \frac{5\pi}{6}$

-2 قابل قبول

ب) $\cos^2 x + \cos x - 2 = 0$

$\cos^2 x - \sin^2 x$

$\cos^2 x - 1 + \cos^2 x + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\cos x = 1 \rightarrow x = 2k\pi$

$\frac{c}{a} = \frac{-3}{2}$ قابل قبول

۲- الف) $\sin^4 x - \cos^4 x = \sin 2x$

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

$1 \quad -\cos 2x = \sin 2x \cos 2x$

$2\sin^2 x \cos^2 x + \cos^2 x = 0 \rightarrow \cos^2 x (2\sin^2 x + 1) = 0$

$\rightarrow \cos 2x = 0 \rightarrow 2x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\sin 2x = \frac{-1}{2} \rightarrow 2x = 2k\pi - \frac{\pi}{6} \rightarrow k\pi - \frac{\pi}{12}$

$2k\pi + \frac{7\pi}{6} \rightarrow k\pi + \frac{7\pi}{12}$

dotnote

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ب) $2\cos^2 2x + 2\sin x \cos x = 1$

$1 + \cos^2 x + \sin^2 x = 1 \longrightarrow \cos^2 x + \sin^2 x = 0$

$\cos^2 x = -\sin^2 x \longrightarrow \sin(-x) = \cos\left(\frac{\pi}{2} - (-x)\right)$

$\cos^2 x = \cos\left(\frac{\pi}{2} + x\right) \longrightarrow x = 2k\pi + \frac{\pi}{2} + x$ غ ق ق

$x = 2k\pi - \frac{\pi}{2} - x$

$2x = 2k\pi - \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{2} - \frac{\pi}{4}$

۳-

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x}$

$\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \longrightarrow \frac{1}{\sin x}(2\cos^2 x + \cos x - 1) = 0$

$\cos x \longrightarrow -1 \longrightarrow x = 2k\pi + \pi$ ق ق ق $(\sin x = 0)$

$\frac{-c}{a} = \frac{1}{2} \longrightarrow x = 2k\pi + \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x \cos x + \cos^2 x = 2$

$2\sin^2 x - 1 = -2\cos^2 x \longrightarrow -\sin x \cos x - \cos x = 0 \longrightarrow -\cos x(\sin x + 1) = 0$

$\cos x = 0 \longrightarrow x = k\pi + \frac{\pi}{2}$

$\sin x = -1 \longrightarrow x = 2k\pi - \frac{\pi}{2}$

$k\pi + \frac{\pi}{2}$

۴-

الف) $\cos\left(\frac{\pi}{4} + x\right)\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{4}$

$\cos(x - \frac{\pi}{4}) - \cos(\frac{\pi}{4} - x)$

$x + \frac{\pi}{4} + x - \frac{\pi}{4} = \frac{\pi}{2} \longrightarrow \cos(\frac{\pi}{2} - x) \sin(x + \frac{\pi}{4})$

$\cos(x + \frac{\pi}{4}) \sin(x + \frac{\pi}{4}) + \frac{1}{2}\sin(2x + \frac{\pi}{2}) = \frac{1}{4}$

$\cos 2x = \frac{1}{2} \longrightarrow 2x = 2k\pi \pm \frac{\pi}{3} \longrightarrow x = k\pi \pm \frac{\pi}{6}$

Date No

آیلا فروزنیا

۱) $\cos(2x-\frac{\pi}{9})=-\sin 2x$

$\cos(2x-\frac{\pi}{9})=\sin(-2x)=\cos(\frac{\pi}{2}+2x) \rightarrow 2x-\frac{\pi}{9}=2k\pi+\frac{\pi}{2}+2x$ غلط

$2x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x \rightarrow 4x=2k\pi-\frac{7\pi}{18} \rightarrow x=\frac{k\pi}{2}-\frac{7\pi}{72}$

۵- $\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \rightarrow \frac{2\sin 2x\cos 2x+\sin 2x}{\sin 2x}$

$\because \neq 0 \rightarrow 2x\neq k\pi \rightarrow x\neq\frac{k\pi}{2}$

$\sin^2 x+\cos^2 x=1$

$2\sin 2x\cos 2x+\sin 2x=\sin 2x$

$\sin 2x=0 \rightarrow 2x=k\pi \rightarrow x=\frac{k\pi}{2}$ غلط

$\cos 2x=0 \rightarrow 2x=k\pi+\frac{\pi}{2} \rightarrow x=\frac{k\pi}{2}+\frac{\pi}{4}$

۶- $\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2}=\frac{1}{\sin\frac{x}{2}}+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$ $\quad 1+\cos\frac{x}{2}\neq 0 \rightarrow \cos\frac{x}{2}\neq -1 \rightarrow \frac{x}{2}\neq 2k\pi+\pi$

$\sin\frac{x}{2}\neq 0 \rightarrow \frac{x}{2}\neq k\pi$

$\tan\frac{x}{4}=\frac{1}{\tan\frac{x}{2}} \rightarrow \tan^2\frac{x}{4}=1 \rightarrow \tan\frac{x}{4}=\pm 1$

$\rightarrow \frac{x}{4}=\frac{k\pi}{2}+\frac{\pi}{4} \rightarrow \frac{x}{2}=k\pi+\frac{\pi}{2} \rightarrow x=2k\pi+\pi$

$-\pi\leq x\leq\frac{3\pi}{2} \rightarrow x=-\pi, \pi$

$x=|\pi-(-\pi)|=2\pi$

آیدا فروغی

۷- $\cos^2 x - \sin^2 x \cos^3 x = 1 \rightarrow \cos^2 x - 1 - \sin^2 x \cos^3 x = 0$

$-\sin^2 x$

$-\sin^2 x (1 + \cos^3 x) = 0$

$-\sin x = 0 \rightarrow x = k\pi \rightarrow x = 0, \pi, 2\pi$

$1 + \cos^3 x = 0 \rightarrow \cos^3 x = -1 \rightarrow 3x = 2k\pi + \pi \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$

$\rightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

جواب‌ها $= 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$ (۵ جواب)

۸- $\tan 3x = \frac{1 - \tan x}{1 + \tan x} \rightarrow \tan 3x = \tan\left(\frac{\pi}{4} - x\right) \rightarrow 3x = k\pi + \frac{\pi}{4} - x \rightarrow 4x = k\pi + \frac{\pi}{4}$

$\frac{1 - \tan x}{1 + \tan x} = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}$

$x = \frac{k\pi}{4} + \frac{\pi}{16}$

۹- $\sqrt{1 + \sin 2x} = \sqrt{2\cos^2 x} \rightarrow \sqrt{1 + 2\sin x \cos x} = \sqrt{2}\cos x \rightarrow |\sin x + \cos x| = \sqrt{2}\cos x$

$\sqrt{2}|\cos x| = \sin x + \cos x \rightarrow \sin x + \cos x \geq 0 \rightarrow \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) \geq 0$

$\sin\left(x + \frac{\pi}{4}\right) \geq 0 \rightarrow 2k\pi \leq x + \frac{\pi}{4} \leq 2k\pi + \pi \rightarrow 2k\pi - \frac{\pi}{4} \leq x \leq 2k\pi + \frac{3\pi}{4}$

$\int_0^{2\pi} |\sin x - \cos x| dx = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x) dx + \int_{\frac{5\pi}{4}}^{2\pi} (\cos x - \sin x) dx$

$= \sqrt{2} + 2\sqrt{2} + \sqrt{2} = 4\sqrt{2}$

$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{2} \rightarrow x - \frac{\pi}{4} = \frac{\pi}{6} + 2k\pi \quad , \quad x - \frac{\pi}{4} = \frac{5\pi}{6} + 2k\pi$

$x = \frac{5\pi}{12} + 2k\pi \quad , \quad x = \frac{13\pi}{12} + 2k\pi$

دو جواب داخل $[0, 2\pi]$

Date No

آیلا فروغنیا

۱۰- الف) $\sin^4 x - \cos^4 x = 1$

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 1$

$\sin^2 x - \cos^2 x = 1$

$-\cos^2 x = 1 \rightarrow \cos 2x = -1 \rightarrow 2x = 2k\pi + \pi$

$x = k\pi + \frac{\pi}{2} \rightarrow \frac{\pi}{2} / \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$

$\sin x = 0 / \cos x = +1 \rightarrow 0, 2\pi$

$\sin x = -1 / \cos x = 0 \rightarrow \frac{3\pi}{2}$

dotnote

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