۲۰ آفرین

نام و نام خانوادگی ...ستاره سعیدزاده... پاسخنامه تشریحی تکلیف شماره ...۱۴... کلاس ...ریاضی دهم A...

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الف) $\frac{1-\cos 2x}{2} = \sin^2 x \to \cos 2x = 1-2\sin^2 x$

$1-2\sin^2 x = 3\sin x - 1$

$2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \frac{-3 \pm \sqrt{25}}{4} \to -2 \ ✗ ,\ \frac{1}{2} \ ✓$

$x = 2k\pi + \frac{\pi}{6}$, $x = 2k\pi + \frac{5\pi}{6}$

ب) $\frac{\cos 2x + 1}{2} = \cos^2 x \to \cos 2x = 2\cos^2 x - 1$

$2\cos^2 x + \cos x - 3 = 0$

$\cos x = \frac{-1 \pm \sqrt{25}}{4} \to -\frac{3}{2} \ ✗ ,\ 1 \ ✓$

$x = 2k\pi$

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الف) $\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x}\underbrace{(\sin^2 x + \cos^2 x)}_{1} = 2\sin 2x\cos 2x$

$2\sin 2x\cos 2x + \cos 2x = 0$

$\cos 2x(2\sin 2x + 1) = 0$

$\cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\sin 2x = -\frac{1}{2} \to \sin(-\frac{\pi}{6}) \to x = k\pi - \frac{\pi}{12}$, $x = k\pi + \frac{7\pi}{12}$

ب) $\underbrace{2\cos^2 x - 1}_{\cos 2x} + \underbrace{2\sin x\cos x}_{\sin 2x} = 0$

$\cos 2x = -\sin 2x = \sin(-2x) = \cos(\frac{\pi}{2} - (-2x))$

$\cos 2x = \cos(\frac{\pi}{2} + 2x) \to 2x = 2k\pi + \frac{\pi}{2} + 2x \ ✗$

$x = \frac{k\pi}{2} - \frac{\pi}{8} \ ✓$

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الف) $\frac{\cos x}{\sin x}(2\cos x + 1) - \frac{1}{\sin x} = 0$

$\frac{1}{\sin x}(2\cos^2 x + \cos x - 1) = 0$

$\sin x \neq 0 \to x \neq k\pi$

$\cos x = -1$, $\cos x = \frac{1}{2}$

$\cos x = -1 \to x = 2k\pi + \pi \ ✗$

$\cos x = \frac{1}{2} \to x = 2k\pi \pm \frac{\pi}{3} \ ✓$

ب) $\underbrace{2\sin^2 x - 2}_{2(\sin^2 x - 1)} - \sin x\cos x + \cos^2 x = 0$

$-\sin x\cos x - \cos x = 0 \to -\cos x(\sin x + 1) = 0$

$\cos x = 0 \to x = k\pi + \frac{\pi}{2}$

$\sin x = -1 \to x = 2k\pi + \frac{3\pi}{2}$

$\boxed{x = k\pi + \frac{\pi}{2}}$

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الف) $\cos(x + \frac{\pi}{4})\cos(x - \frac{\pi}{4}) = \frac{1}{4}$

$\cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} - x)$

$x + \frac{\pi}{4} + \frac{\pi}{4} - x = \frac{\pi}{2} \to \cos(\frac{\pi}{4} - x) = \sin(x + \frac{\pi}{4})$

$\cos(x + \frac{\pi}{4})\sin(x + \frac{\pi}{4}) = \frac{1}{2}\sin(2x + \frac{\pi}{2}) = \frac{1}{4}$

$\cos 2x = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = \sin(-2x) = \cos(\frac{\pi}{2} + 2x)$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \ ✗$

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \Rightarrow 4x = 2k\pi - \frac{\pi}{2} + \frac{\pi}{9}$

$x = \frac{k\pi}{2} - \frac{7\pi}{72}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x \Rightarrow \frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x}$, $\sin^2 x + \cos^2 x = 1$

$2\sin 2x\cos 2x + \sin 2x = \sin 2x$

$2\sin 2x\cos 2x = 0$

$\sin 2x \neq 0$, $2x \neq k\pi \to x \neq \frac{k\pi}{2}$ غیرقابل قبول

$\sin 2x = 0 \ ✗$

$\cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

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$$\frac{\sin 2x}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$$

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \tan\frac{x}{4} = \frac{1}{\tan\frac{x}{4}} \to \tan^2\frac{x}{4} = 1 \to \tan\frac{x}{4} = \pm 1$$ ۲ ۶

$\cos\frac{x}{2} \neq -1$ $\sin\frac{x}{2} \neq 0$

$\frac{x}{2} \neq 2k\pi+\pi$, $\frac{x}{2} \neq k\pi$

$$\frac{x}{4} = \frac{k\pi}{2} + \frac{\pi}{4} \to \frac{x}{2} = k\pi + \frac{\pi}{2} \to x = 2k\pi + \pi,\ -\pi \le x \le \frac{3\pi}{2} \to x = \pi, (-\pi)$$

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$$\underbrace{\cos^2 x - 1}_{-\sin^2 x} - \sin^2 x\cos 3x = 0 \Rightarrow -\sin^2 x(1+\cos 3x) = 0$$

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$\sin x = 0 \to x = k\pi \to x = 0, \pi, 2\pi$

غلط

$$1+\cos 3x = 0 \to \cos 3x = -1 \to 3x = 2k\pi + \pi \to x = \frac{2k\pi}{3} + \frac{\pi}{3} \to x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$ ۷

$\tan(\frac{\pi}{4} - x) = \tan 2x$

$$\frac{1-\tan x}{1+\tan x} = \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\tan x} \to 2x = k\pi + \frac{\pi}{4} - x$$ ۸

$3x = k\pi + \frac{\pi}{4}$ ۲

$x = \frac{k\pi}{3} + \frac{\pi}{12}$ ✓

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \to \sqrt{1+2\sin x\cos x} = \sqrt{2}(\cos^2 x - \sin^2 x) \to |\cos x + \sin x| = \sqrt{2}(\cos^2 x - \sin^2 x)$$

$$\xrightarrow{\sin x \neq -\cos x} \sqrt{2}\,|\cos x - \sin x| = 1 \to \sqrt{2}\,|\sin x - \cos x| = 1$$

$\sin x - \cos x = \sqrt{2}(\sin(x - \frac{\pi}{4}))$ $\sqrt{2}\,|\sqrt{2}\sin(x-\frac{\pi}{4})| = 1 \to \overset{2}{\cancel{\sqrt{2}\times\sqrt{2}}}\,|\sin(x-\frac{\pi}{4})| = 1$ ۲ ۹

$$|\sin(x-\frac{\pi}{4})| = \frac{1}{2} \to \sin(x-\frac{\pi}{4}) = \pm\frac{1}{2} \to x - \frac{\pi}{4} = k\pi \pm \frac{\pi}{6} \to x = k\pi \pm \frac{\pi}{6} + \frac{\pi}{4} \to$$ ۰/۲۵

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الف) $\sin^4 x - \cos^4 x = 1$

$\underbrace{(\sin^2 x - \cos^2 x)}_{1}\underbrace{(\sin^2 x + \cos^2 x)}_{1} = 1$

$-\cos 2x = 1 \to \cos 2x = 1$

$2x = 2k\pi + \pi$

$x = k\pi + \frac{\pi}{2} \to \frac{\pi}{2}, \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$

با امتحانات زوایای راحت حل

$x = 0 \to -1$ ✓

$x = \frac{\pi}{2} \to 1$ ✗ ۲ ۱۰

$x = \pi \to 1$ ✗

$x = \frac{3\pi}{2} \to -1$ ✓

$x = 2\pi \to -1$ ✓

$\boxed{2\pi, \frac{3\pi}{2}, 0}$