

۱) $\frac{1 - \cos^2 u}{v} = \sin^2 u \rightarrow \cos^2 u = 1 - \sin^2 u$

$1 - \sin^2 u = \sin^2 u - 1$

$v \sin^2 u + \sin^2 u = v + 1$

$\sin u = \frac{-v \pm \sqrt{v^2 + 4}}{2}$

$\begin{cases} u = \arccos \left(\frac{-v \pm \sqrt{v^2 + 4}}{2} \right) \\ u = \arcsin \left(\frac{-v \pm \sqrt{v^2 + 4}}{2} \right) \end{cases}$

۲) $\frac{\cos^2 u + 1}{v} = \cos^2 u \rightarrow \cos^2 u = \cos^2 u - 1$

$v \cos^2 u + \cos^2 u = v + 1$

$\cos u = \frac{-1 \pm \sqrt{1 + v}}{2}$

$\begin{cases} u = \arccos \left(\frac{-1 \pm \sqrt{1 + v}}{2} \right) \\ u = \arcsin \left(\frac{-1 \pm \sqrt{1 + v}}{2} \right) \end{cases}$

۳) $\frac{(\sin^2 u - \cos^2 u)}{-\cos^2 u} = \frac{(\sin^2 u + \cos^2 u)}{1} = \sin^2 u \cos^2 u$

$\sin^2 u \cos^2 u + \cos^2 u = 0$

$\cos^2 u (\sin^2 u + 1) = 0$

$\cos^2 u = 0 \rightarrow u = \frac{\pi}{2} + k\pi \rightarrow u = \frac{\pi}{2} + k\pi$

$\sin^2 u = 1 \rightarrow \sin u = \pm 1 \rightarrow u = \frac{\pi}{2} + k\pi \rightarrow u = \frac{\pi}{2} + k\pi$

۴) $\frac{v \cos^2 u - 1 + \sin^2 u \cos^2 u}{\cos^2 u + \sin^2 u} = 0$

$\cos^2 u = -\sin^2 u \rightarrow \sin^2 u = \cos^2 u \rightarrow \sin u = \pm \cos u$

$\cos u = \cos \left(\frac{\pi}{2} + k\pi \right) \rightarrow u = \frac{\pi}{2} + k\pi$

۵) $\frac{\cos u}{\sin u} (v \cos u + 1) = \frac{1}{\sin u} = 0$

۶) $\frac{v \sin^2 u - 1 - \sin^2 u \cos^2 u + \cos^2 u}{v (\sin^2 u)} = 0$

$\sin^2 u \cos^2 u + \cos^2 u = 0 \rightarrow \cos^2 u (\sin^2 u + 1) = 0$

$\frac{1}{\sin u} (v \cos^2 u + \cos^2 u - 1) = 0 \rightarrow \cos^2 u = 1 \rightarrow u = k\pi$

$\sin u = 0 \rightarrow u = k\pi$

$\cos u = -1 \rightarrow \cos u = \frac{1}{v}$

$\cos u = 0 \rightarrow u = \frac{\pi}{2} + k\pi$

$\sin u = 1 \rightarrow u = \frac{\pi}{2} + k\pi$

$u = k\pi + \frac{\pi}{2}$

۷) $\cos \left(u + \frac{\pi}{2} \right) \cos \left(u - \frac{\pi}{2} \right) = \frac{1}{2}$

$\cos \left(u - \frac{\pi}{2} \right) = \cos \left(\frac{\pi}{2} - u \right)$

$u + \frac{\pi}{2} = \frac{\pi}{2} - u \rightarrow u = 0 \rightarrow \cos \left(\frac{\pi}{2} - u \right) = \cos \left(\frac{\pi}{2} \right) = 0$

$\cos \left(u + \frac{\pi}{2} \right) \sin \left(u + \frac{\pi}{2} \right) = \frac{1}{v} \sin \left(u + \frac{\pi}{2} \right) = \frac{1}{v}$

$\cos^2 u = \frac{1}{v} \rightarrow u = \arccos \left(\frac{1}{v} \right) \rightarrow u = \arccos \left(\frac{1}{v} \right)$

۸) $\cos \left(u - \frac{\pi}{4} \right) = \sin \left(-u \right) \cos \left(\frac{\pi}{4} + u \right)$

$u - \frac{\pi}{4} = \frac{\pi}{4} + u \rightarrow u = \frac{\pi}{2}$

$u - \frac{\pi}{4} = \frac{\pi}{4} - u \rightarrow u = \frac{\pi}{2}$

$u = \frac{\pi}{2}$

$\frac{\sin^2 u - \sin^2 u}{\sin^2 u} = \frac{\sin^2 u - \sin^2 u}{\sin^2 u} \Rightarrow \frac{\sin^2 u \cos^2 u + \sin^2 u}{\sin^2 u} = 1$

$\sin^2 u \cos^2 u + \sin^2 u = \sin^2 u$

$\sin^2 u \cos^2 u = 0$

$\sin^2 u = 0$

$\cos^2 u = 0 \rightarrow u = \frac{\pi}{2} + k\pi \rightarrow u = \frac{\pi}{2} + k\pi$

$$\frac{\sin \mu}{1 + \cos \frac{\mu}{p}} = \frac{1}{\sin \frac{\mu}{p}} + \cot \frac{\mu}{p} = \frac{1}{\sin \frac{\mu}{p}} + \frac{\cos \frac{\mu}{p}}{\sin \frac{\mu}{p}} = \frac{1 + \cos \frac{\mu}{p}}{\sin \frac{\mu}{p}}$$

$$\frac{\sin \frac{\mu}{p}}{1 + \cos \frac{\mu}{p}} = \frac{1 + \cos \frac{\mu}{p}}{\sin \frac{\mu}{p}} \rightarrow \tan \frac{\mu}{2} = \frac{1}{\tan \frac{\mu}{2}} \rightarrow \tan^2 \frac{\mu}{2} = 1 \rightarrow \tan \frac{\mu}{2} = \pm 1$$

$$\begin{aligned} \cos \frac{\mu}{p} \neq -1 \quad \sin \frac{\mu}{p} \neq 0 \\ \mu \neq p\pi + \pi \quad \frac{\mu}{p} \neq \pi \end{aligned} \quad \frac{\mu}{2} = \frac{k\pi}{p} + \frac{\pi}{2} \rightarrow \frac{\mu}{p} = k\pi + \frac{\pi}{p} \rightarrow \mu = k\pi + \frac{\pi}{p} \rightarrow \mu = k\pi + \frac{\pi}{p} \rightarrow \mu = k\pi + \frac{\pi}{p}$$

$$\frac{\cos^2 \mu - 1}{-\sin^2 \mu} = \frac{\sin^2 \mu \cos^2 \mu}{-\sin^2 \mu} = -\sin^2 \mu (1 + \cos^2 \mu)$$

$$\sin \mu = 0 \rightarrow \mu = k\pi \rightarrow \mu = 0, \pi, 2\pi$$

$$1 + \cos^2 \mu = 0 \rightarrow \cos^2 \mu = -1 \rightarrow \mu = k\pi + \frac{\pi}{2} \rightarrow \mu = \frac{k\pi}{2} + \frac{\pi}{2} \rightarrow \mu = \frac{k\pi}{2} + \frac{\pi}{2}$$

$$\tan\left(\frac{\pi}{2} - \mu\right) = \tan \mu$$

$$\frac{1 - \tan \mu}{1 + \tan \mu} = \frac{\tan \frac{\pi}{2} - \tan \mu}{1 + \tan \frac{\pi}{2} \tan \mu} \rightarrow \mu = k\pi + \frac{\pi}{2} - \mu$$

$$\mu = k\pi + \frac{\pi}{2}$$

$$\mu = \frac{k\pi}{2} + \frac{\pi}{4} \checkmark$$

$$\sqrt{1 + \sin^2 \mu} = \sqrt{1 + \cos^2 \mu} \rightarrow \sqrt{1 + \sin^2 \mu} = \sqrt{1 + \cos^2 \mu} \rightarrow \sqrt{1 + \sin^2 \mu} = \sqrt{1 + \cos^2 \mu} \rightarrow \sqrt{1 + \sin^2 \mu} = \sqrt{1 + \cos^2 \mu}$$

$$\frac{\sin^2 \mu + \cos^2 \mu}{\sqrt{1 + \sin^2 \mu} \sqrt{1 + \cos^2 \mu}} = 1 \rightarrow \sqrt{1 + \sin^2 \mu} \sqrt{1 + \cos^2 \mu} = 1$$

$$\sin^2 \mu + \cos^2 \mu = \sqrt{1 + \sin^2 \mu} \sqrt{1 + \cos^2 \mu} \rightarrow \sqrt{1 + \sin^2 \mu} \sqrt{1 + \cos^2 \mu} = 1$$

$$\left| \sin\left(\mu - \frac{\pi}{2}\right) \right| = \frac{1}{2} \rightarrow \sin\left(\mu - \frac{\pi}{2}\right) = \pm \frac{1}{2} \rightarrow \mu - \frac{\pi}{2} = k\pi \pm \frac{\pi}{2} \rightarrow \mu = k\pi + \frac{\pi}{2} \pm \frac{\pi}{2} \rightarrow \mu = k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}$$

$$\cos \mu = \sin \mu$$

$$\left(\sin^2 \mu - \cos^2 \mu \right) \left(\sin^2 \mu + \cos^2 \mu \right) = 1$$

$$-\cos^2 \mu = 1 \rightarrow \cos^2 \mu = -1$$

$$\mu = k\pi + \pi$$

$$\mu = k\pi + \frac{\pi}{2} \rightarrow \mu = \frac{k\pi}{2} + \frac{\pi}{2}$$

$$\sin^2 \mu - \cos^2 \mu = 1$$

$$\mu = 0, \pi, 2\pi$$

$$\mu = 0 \rightarrow -1 \checkmark$$

$$\mu = \pi \rightarrow 1 \checkmark$$

$$\mu = 2\pi \rightarrow 1 \checkmark$$

$$\mu = \frac{\pi}{2} \rightarrow -1 \checkmark$$

$$\mu = \frac{3\pi}{2} \rightarrow -1 \checkmark$$

$$\mu = \frac{k\pi}{2} + \frac{\pi}{2}$$