مبینا رضایی

۱) الف) $\cos 2x = 3\sin x - 1 \xrightarrow{\text{طلایی}} 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\xrightarrow{\text{ددسی}} \sin^2 x + 3\sin - 4 = 0$

- $\sin x = \frac{1}{2} \longrightarrow \underline{x = 2k\pi + \frac{\pi}{6} \,,\, 2k\pi + \frac{5\pi}{6}}$
- $\sin x = -2$ غ ق ق

ب) $\cos 2x + \cos x - 2 = 0 \xrightarrow{\text{طلایی}} 2\cos^2 x + \cos x - 3 = 0$

- $\cos x = +1 \rightarrow \underline{x = 2k\pi}$
- $\cos x = -\frac{3}{2}$ غ ق ق

۲)

الف) $\cancel{(\sin^2 x + \cos^2 x)}(\sin^2 x - \cos^2 x) = \sin 4x \longrightarrow -\cos 2x = \sin 4x$

$\longrightarrow \sin\left(\frac{3\pi}{2} + 2x\right) = \sin 4x$

- $\rightarrow 4x = 2k\pi + \frac{3\pi}{2} + 2x$
- $\rightarrow 4x = 2k\pi + \pi - \frac{3\pi}{2} - 2x$

$x = k\pi + \frac{3\pi}{4}$ و $x = \frac{k\pi}{3} - \frac{\pi}{2}$

ب) $\underbrace{2\cos^2 x}_{\text{طلایی}} + 2\sin x\cos x = 1 \longrightarrow \cancel{1} + \cos 2x + \sin 2x = \cancel{1}$

$\longrightarrow \cos 2x = -\sin 2x \longrightarrow \cos 2x = \cos\left(\frac{\pi}{2} + 2x\right)$

- $2x = 2k\pi + \frac{\pi}{2} + 2x$ ✗
- $2x = 2k\pi - \frac{\pi}{2} - 2x$

$\longrightarrow \underline{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

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۳)

الف) $\dfrac{\cos x}{\sin x}(2\cos x+1)=\dfrac{1}{\sin x} \xrightarrow{\sin x\neq 0} 2\cos^2 x+\cos x-1=0 \rightarrow \cos x=-1$ ← به خاطر شرط‌ها رد می‌شود

$\rightarrow \cos x=\dfrac{1}{2}$

$\longrightarrow x=2k\pi \pm \dfrac{\pi}{3}$

ب) $\underbrace{2\sin^2 x-2}_{-2\cos^2 x}+\cos^2 x-\sin x\cos x=0 \longrightarrow -\cos^2 x-\sin x\cos x=0$

$\longrightarrow -\cos x(\cos x+\sin x)=0$

$\rightarrow \cos x=0 \rightarrow x=k\pi+\dfrac{\pi}{2}$

$\rightarrow \cos x=-\sin x \longrightarrow x=k\pi-\dfrac{\pi}{4}$

۴)

الف) $\cos(\dfrac{\pi}{4}+x)\times\cos(\dfrac{\pi}{4}-x)=\dfrac{1}{4} \xrightarrow{\text{مجموع زوایا } \frac{\pi}{2}}$

$\longrightarrow \cos(\dfrac{\pi}{4}+x)\times\sin(\dfrac{\pi}{4}+x)=\dfrac{1}{4} \longrightarrow \dfrac{1}{2}\sin(\dfrac{\pi}{2}+2x)=\dfrac{1}{4}$

$\longrightarrow \cos 2x=\dfrac{1}{2} \longrightarrow 2x \longrightarrow 2k\pi\pm\dfrac{\pi}{3} \longrightarrow x=k\pi\pm\dfrac{\pi}{6}$

۵)

$\dfrac{\sin 4x+\sin 2x}{\sin 2x}=1 \qquad \sin 2x\neq 0$

$\sin 4x+\sin 2x=\sin 2x \longrightarrow \sin 4x=0$

$\sin 4x=2\sin 2x\cos 2x \xrightarrow{\sin 2x\neq 0} \cos 2x=0$

$\longrightarrow 2x=k\pi+\dfrac{\pi}{2} \longrightarrow x=\dfrac{k\pi}{2}+\dfrac{\pi}{4}$

۶)

$\dfrac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\dfrac{\cos\frac{x}{2}+1}{\sin\frac{x}{2}} \xrightarrow[\sin\frac{x}{2}\neq 0,\ \cos\frac{x}{2}\neq -1]{\tan\theta=\frac{\sin 2\theta}{1+\cos 2\theta}} \tan\dfrac{x}{4}=\cot\dfrac{x}{4}$

$\longrightarrow \dfrac{x}{4}=\dfrac{k\pi}{2}+\dfrac{\pi}{4} \longrightarrow x=2k\pi+\pi \longrightarrow x=-\pi,\pi$

$-\pi\leq x\leq\dfrac{3\pi}{2}$

$\pi-(-\pi)=2\pi$

$\cos^2 x - \sin^2 x \cos 3x = 1 \longrightarrow \underbrace{\cos^2 x - 1}_{-\sin^2 x} - \sin^2 x \cos 3x = 0$ (۷

$\sin^2 x + \sin^2 x \cos 3x = 0 \longrightarrow \sin^2 x (1 + \cos 3x) = 0$

$\sin^2 x = 0 \longrightarrow \sin x = 0$

$1 + \cos 3x = 0 \longrightarrow \cos 3x = -1 \longrightarrow 3x = 2k\pi + \pi \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$

جواب‌ها : $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi \longrightarrow$ ۵ جواب

(۸

$\tan(\frac{\pi}{4} - x) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = \frac{1 - \tan x}{1 + \tan x}$

$\longrightarrow \tan(\frac{\pi}{4} - x) = \tan 3x \longrightarrow 3x = k\pi + \frac{\pi}{4} - x$

$x = k\pi + \frac{\pi}{16}$

$\sqrt{\underbrace{1}_{\sin^2 x + \cos^2 x} + 2\sin x \cos x} = \sqrt{(\sin x + \cos x)^2} = |\sin x + \cos x| = \sqrt{2}\cos 2x$ (۹

$|\sin x + \cos x| = \sqrt{2}(\cos x - \sin x)(\cos x + \sin x) \xrightarrow{\sin x + \cos x \neq 0}$

$\sqrt{2}\,\overbrace{|\cos x - \sin x|}^{}_{|\sin x - \cos x|} = 1 \longrightarrow \sqrt{2}\,|\sqrt{2}\sin(x - \frac{\pi}{4})| = 1$ $|\sin$

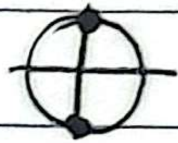
$\longrightarrow 2|\sin(x - \frac{\pi}{4})| = 1 \longrightarrow |\sin(x - \frac{\pi}{4})| = \frac{1}{2} \begin{cases} \sin(x - \frac{\pi}{4}) = \frac{1}{2} \\ \sin(x - \frac{\pi}{4}) = -\frac{1}{2} \end{cases}$

$x - \frac{\pi}{4} = k\pi \pm \frac{\pi}{6} \longrightarrow x = k\pi \pm \frac{\pi}{6} - \frac{\pi}{4} \xrightarrow{[0, \pi]}$ ۲ جواب

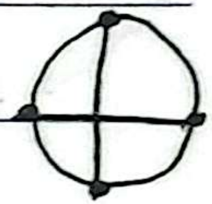
۱۰)

الف) $(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)=\overset{\sin^2+\cos^2}{\uparrow}1 \longrightarrow \sin^2 x-\cos^2 x=\sin^2 x+\cos^2 x$

$\longrightarrow 2\cos^2 x=0 \longrightarrow \cos x=0 \longrightarrow x=\frac{\pi}{2}$ و $\frac{3\pi}{2}$



ب) $\sin^3 x-\cos^3 x=-1 \longrightarrow$ نقاط اصلی را چک می‌کنیم



$x=0$ ✓ $\quad x=\frac{\pi}{2}$ ✗ $\quad x=\pi$ ✗ $\quad x=\frac{3\pi}{2}$ ✓ $\quad x=2\pi$ ✓

جواب‌ها $\longrightarrow 0$ و $\frac{3\pi}{2}$ و 2π