A دوازدهم دختر — تکلیف شماره ۱۶ — مرسا موسوی

۱۹ آفرین

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۱) الف) $\cos 2x = 3\sin x - 1 \rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow \sin^2 x + 3\sin x - 4 = 0 \rightarrow$

$(\sin x + 4)(\sin x - 1) = 0 \longrightarrow x = \left\{2k\pi + \frac{\pi}{6},\ 2k\pi + \frac{5\pi}{6}\right\}$

$\times\ \frac{-2}{2}$ $\quad \frac{1}{2}\checkmark$

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ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0$ $\begin{cases} \cos x = 1 \ \checkmark \\ \cos x = -\frac{3}{2} \ \times \end{cases} \rightarrow x = 2k\pi$

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۲) الف) $\sin^4 x - \cos^4 x = \sin 4x \rightarrow \sin^2 x - \cos^2 x = 2\sin 2x \cos 2x \rightarrow \sin 2x = -\frac{1}{2} \rightarrow$

$2x = \left\{2k\pi - \frac{\pi}{6},\ 2k\pi - \frac{5\pi}{6}\right\} \rightarrow x = \left\{k\pi - \frac{\pi}{12},\ k\pi - \frac{5\pi}{12}\right\}$

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ب) $2\cos^2 \alpha - 1 + 2\sin\alpha\cos\alpha = 0 \rightarrow \cos 2\alpha + \sin 2\alpha = 0 \rightarrow$  $\rightarrow 2\alpha = k\pi + \frac{3\pi}{4} \rightarrow$

$\alpha = \frac{k\pi}{2} + \frac{3\pi}{8}$

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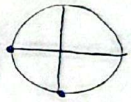
۳) الف) $\cot x(2\cos x + 1) = \frac{1}{\sin\alpha} \rightarrow \frac{\cos x(2\cos x + 1)}{\sin x} = \frac{1}{\ } \rightarrow 2\cos^2 x + \cos x - 1 = 0 \rightarrow$ ۱٫۷۵

$\cos x = -1$ یا $\frac{1}{2} \rightarrow x = \left\{\cancel{2k\pi + \pi},\ 2k\pi \pm \frac{\pi}{3}\right\}$

اگر $\cos x = -1$ باشد، $\sin x = 0$ است و عبارت تعریف نشده می‌شود

پس جواب غیر قابل قبول محسوب می‌شود

ب) $2\sin x - \sin x\cos x + \cos^2 x = 2 \rightarrow \sin^2 x - 1 - \sin x\cos x = 0 \rightarrow \sin^2 x - 1 = \sin x\cos x \xrightarrow{\times 2}$

$2\sin^2 x - 1 - 1 = 2\sin x\cos x \rightarrow -\cos 2x = \sin 2x + 1 \rightarrow \sin 2x + \cos 2x = -1 \rightarrow$ 

$2x = \left\{2k\pi + \pi,\ 2k\pi + \frac{3\pi}{2}\right\} \rightarrow x = \left\{k\pi + \frac{\pi}{2},\ k\pi + \frac{3\pi}{4}\right\}$

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۴) الف) $\cos\left(\frac{\pi}{4} + x\right)\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{2} \rightarrow 2\sin\left(x + \frac{\pi}{4}\right)\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{2} \rightarrow \sin\left(2x + \frac{\pi}{2}\right) = \frac{1}{2}$

$\cos 2x = \frac{1}{2} \rightarrow 2x = \left\{2k\pi \pm \frac{\pi}{3}\right\} \rightarrow x = \left\{k\pi \pm \frac{\pi}{6}\right\}$

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ب) $\cos\left(2x - \frac{\pi}{9}\right) = \cos\left(2x + \frac{\pi}{2}\right) \rightarrow 2x - \frac{\pi}{9} = 2k\pi \pm \left(2x + \frac{\pi}{2}\right) \rightarrow$ $\times$ $\checkmark$

$4x = 2k\pi - \frac{\pi}{2} + \frac{\pi}{9} \rightarrow x = \frac{(36k - 7)\pi}{72}$

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۵) $\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x \rightarrow 2\cos 2x + 1 = \cos^2 x + \sin^2 x \rightarrow$

$2(\cos^2 x - \sin^2 x) + 1 = \cos^2 x + \sin^2 x \rightarrow \cos^2 x - 3\sin^2 x + 1 = 0 \rightarrow 4\sin^2 x = 2 \rightarrow \sin^2 x = \frac{1}{2} \rightarrow$

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$\sin x = \frac{\sqrt{2}}{2} \rightarrow x = \left\{2k\pi + \frac{\pi}{4},\ 2k\pi + \frac{3\pi}{4}\right\}$

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۶) $\frac{\sin\alpha}{1 + \cos\alpha} = \frac{1 + \cos\alpha}{\sin\alpha} \rightarrow \sin^2\alpha = \cos^2\alpha + 1 + 2\cos\alpha \rightarrow 2\cos^2\alpha + 2\cos\alpha = 0 \rightarrow$

$2\cos\alpha(\cos\alpha + 1) = 0 \rightarrow \cos\alpha = \begin{cases} 0 \\ -1 \end{cases} \rightarrow \cos\frac{\alpha}{2} = \{0, -1\}$

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در بازه‌ی مشخص شده فقط π و $-\pi$ قابل قبول $\rightarrow |-\pi - \pi| = 2\pi$

۷)
$\cos^2 x - \sin^2 x\cos 3x = 1 \longrightarrow \sin^2 x\cos 3x = \cancel{\cos^2 x - 1}_{-\sin^2 x} \longrightarrow \sin^2 x(\cos 3x + 1) = 0 \longrightarrow$

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$\begin{cases}\sin^2 x = 0\\ \cos 3x = -1\end{cases} \longrightarrow x = \{k\pi\}$ یا $3x = \{2k\pi + \pi\} \longrightarrow x = \{\frac{2k\pi}{3} + \frac{\pi}{3}\}$

اشتراک جواب‌ها : $x = \{\frac{2k\pi}{3} + \frac{\pi}{3}, k\pi\}$

در بازه‌ی $[0, 2\pi]$ ؟

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۸)
$\frac{1-\tan x}{1+\tan x} = \tan 3x \longrightarrow \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = \tan 3x \longrightarrow$

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$\tan(\frac{\pi}{4} - x) = \tan 3x \longrightarrow 3x = k\pi + \frac{\pi}{4} - x \to 4x = k\pi + \frac{\pi}{4} \longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$

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۹)
$\sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x} = \sqrt{2}(\cos^2 x - \sin^2 x) \longrightarrow |\sin x + \cos x| = \sqrt{2}(\sin x + \cos x)(\cos x - \sin x)$

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if: $\sin x + \cos x \geq 0 \longrightarrow \sqrt{2}\cos x = \sqrt{2}\sin x \longrightarrow \sin x = \cos x \longrightarrow x = \{2k\pi + \frac{\pi}{4}\}$

if: $\sin x + \cos x < 0 \longrightarrow \sqrt{2}\cos x = \sqrt{2}\sin x \longrightarrow \sin x = \cos x \longrightarrow x = \{2k\pi - \frac{3\pi}{4}\}$

$\xrightarrow{\cap} x = \{k\pi + \frac{\pi}{4}\}$

در بازه‌ی $[0, \pi]$ ؟

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۱۰)
الف) $\sin^4 x - \cos^4 x = 1 \longrightarrow \sin^2 x - \cos^2 x = 1 \longrightarrow -\cos 2x = 1 \longrightarrow \cos 2x = -1 \longrightarrow$

$2x = \{2k\pi + \pi\} \longrightarrow x = \{k\pi + \frac{\pi}{2}\} \longrightarrow$ بین این ۲ بازه ۲ جواب داریم.

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ب) $\sin^3 x - \cos^3 x = -1 \longrightarrow \cos^3 x = 1, \sin^3 x = 0 \longrightarrow x = 0, 2\pi$

$\hookrightarrow \cos^3 x = 0, \sin^3 x = -1 \longrightarrow x = \frac{3\pi}{2}$

$\longrightarrow x = \{0, \frac{3\pi}{2}, 2\pi\}$

۳ جواب در این بازه داریم

سوال ۹

$\sqrt{1 + \sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1 + \sin 2x = 2\cos^2 2x$

$1 + \sin 2x = 2 - 2\sin^2 2x \longrightarrow 2\sin^2 2x + \sin 2x - 1 = 0$ $\begin{cases}\sin 2x = -1\\ \sin 2x = +\frac{1}{2}\end{cases}$

$\sin 2x = -1 \longrightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \leq x \leq \pi} x = \frac{3\pi}{4}$

$\sin 2x = \frac{1}{2} \longrightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \leq x \leq \pi} x = \frac{\pi}{12}$ یا $\frac{5\pi}{12}$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود و چون جواب رادیکال است غ ق ق است

* معادله ۲ جواب دارد

سوال ۷

$$\cos^2 x - \sin^2 x \cos 3x = 1 \xrightarrow{\cos^2 x = 1 - \sin^2 x}$$

$$\cancel{1} - \sin^2 x - \sin^2 x \cos 3x = \cancel{1} \longrightarrow \sin^2 x (1 + \cos 3x) = 0 \begin{cases} \sin x = 0 \\ \cos 3x = -1 \end{cases}$$

$$\sin x = 0 \longrightarrow x = k\pi \longrightarrow x = \overset{3}{0}, \overset{2}{\pi}, \overset{1}{2\pi}$$

$$\cos 3x = -1 \longrightarrow 3x = 2k\pi + \pi \longrightarrow x = \frac{2k\pi + \pi}{3} \longrightarrow x = \overset{5}{\frac{\pi}{3}}, \pi, \overset{4}{\frac{5\pi}{3}}$$

معادله ۵ جواب دارد