

الف) $\cos^2 x = 3 \sin x - 1 \rightarrow 1 - \sin^2 x = 3 \sin x - 1 \rightarrow \sin^2 x + \sin x - 2 = 0 \rightarrow$
 $(\sin x + 2)(\sin x - 1) = 0 \rightarrow \sin x = 1 \rightarrow x = 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{5\pi}{2}$
 $x = \frac{\pi}{2} + 2k\pi$

ب) $\cos^2 x + \cos x - 2 = 0 \rightarrow \cos^2 x + \cos x - 2 = 0 \rightarrow \cos x = 1 \vee \cos x = -2$
 $\cos x = 1 \rightarrow x = 2k\pi$

الف) $\sin^2 x - \cos^2 x = \sin x \rightarrow \sin^2 x - \cos^2 x = \sin x \rightarrow \sin^2 x - 1 + \sin x = 0 \rightarrow \sin^2 x + \sin x - 1 = 0$
 $\sin x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$
 $x = \arcsin\left(\frac{-1 \pm \sqrt{5}}{2}\right)$

ب) $\cos^2 x - 1 + \sin x \cos x = 0 \rightarrow \cos^2 x + \sin x \cos x - 1 = 0 \rightarrow \cos x (\cos x + \sin x) = 1$
 $\cos x = 1 \rightarrow x = 2k\pi$
 $\cos x + \sin x = \frac{1}{\cos x} \rightarrow \sin x = \frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$
 $\sin x \cos x = \sin^2 x \rightarrow \cos x = \sin x \rightarrow x = k\pi + \frac{\pi}{4}$

الف) $\cot x (\cos x + 1) = \frac{1}{\sin x} \rightarrow \frac{\cos x (\cos x + 1)}{\sin x} = \frac{1}{\sin x} \rightarrow \cos^2 x + \cos x - 1 = 0$
 $\cos x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$
 $x = \arccos\left(\frac{-1 \pm \sqrt{5}}{2}\right)$

ب) $\sin^2 x - \sin x \cos x + \cos^2 x = 1 \rightarrow \sin^2 x - \sin x \cos x + \cos^2 x = 1 \rightarrow \sin^2 x - 1 = \sin x \cos x$
 $\sin^2 x - 1 = \sin x \cos x \rightarrow \cos^2 x = \sin x + 1 \rightarrow \sin^2 x + \cos^2 x = -1 \rightarrow \sin^2 x = -2$
 $x = \arcsin(\pm \sqrt{-2})$

الف) $\cos\left(\frac{\pi}{4} + x\right) \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{2} \rightarrow \frac{1}{2} [\cos\left(\frac{\pi}{4} + x + x - \frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4} + x - x + \frac{\pi}{4}\right)] = \frac{1}{2}$
 $\frac{1}{2} [\cos(2x) + \cos(\frac{\pi}{2})] = \frac{1}{2} \rightarrow \cos(2x) = 1 \rightarrow 2x = 2k\pi \rightarrow x = k\pi$

ب) $\cos\left(x - \frac{\pi}{4}\right) = \cos\left(x + \frac{\pi}{4}\right) \rightarrow x - \frac{\pi}{4} = 2k\pi \pm \left(x + \frac{\pi}{4}\right) \rightarrow x - \frac{\pi}{4} = x + \frac{\pi}{4} + 2k\pi$
 $x = \frac{\pi}{2} + 2k\pi$

$\frac{\sin^2 x \cos^2 x + \sin^2 x}{\sin^2 x} = \sin^2 x + \cos^2 x \rightarrow \cos^2 x + 1 = \sin^2 x + \cos^2 x + 1 \rightarrow \sin^2 x = 1 \rightarrow \sin x = \pm 1$
 $x = \frac{\pi}{2} + k\pi$

$\cos^2 x - \sin^2 x = 1 \rightarrow \cos^2 x - 1 + \sin^2 x = 1 \rightarrow \sin^2 x = 1 \rightarrow \sin x = \pm 1$
 $x = \frac{\pi}{2} + k\pi$

$\frac{\sin x}{1 + \cos x} = \frac{1 + \cos x}{\sin x} \rightarrow \sin^2 x = \cos^2 x + 1 + \cos x \rightarrow \cos^2 x + \cos x + 1 = 0$
 $\cos x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$

$\cos x (1 + \cos x) = 0 \rightarrow \cos x = 0 \rightarrow x = \frac{\pi}{2} + k\pi$

$-\pi, \pi$ (بداية ونهاية الدائرة) $\rightarrow |-\pi - \pi| = 2\pi$

$$\cos^2 x - \sin^2 x \cos^2 x = 1 \rightarrow \sin^2 x \cos^2 x = \frac{\cos^2 x - 1}{-\sin^2 x} \rightarrow \sin^2 x (\cos^2 x + 1) = 0 \rightarrow \quad (V)$$

$$\begin{cases} \sin^2 x = 0 \\ \cos^2 x = -1 \end{cases} \rightarrow x = \{k\pi\} \quad \underline{\text{L}} \quad x = \{2k\pi + \pi\} \rightarrow x = \left\{ \frac{2k\pi}{2} + \frac{\pi}{2} \right\}$$

استدراجاً : $x = \left\{ \frac{2k\pi}{2} + \frac{\pi}{2}, k\pi \right\}$

$$\frac{1 - \tan x}{1 + \tan x} = \tan \frac{\pi}{4} \rightarrow \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} = \tan \frac{\pi}{4} \rightarrow \quad (A)$$

$$\tan \left(\frac{\pi}{4} - x \right) = \tan \frac{\pi}{4} \rightarrow \frac{\pi}{4} - x = k\pi + \frac{\pi}{4} - x \rightarrow x = k\pi + \frac{\pi}{4} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{4}$$

$$\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x} = \sqrt{2} (\cos^2 x - \sin^2 x) \rightarrow |\sin x + \cos x| = \sqrt{2} (\sin x + \cos x) \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right) \quad (9)$$

$$\begin{aligned} \text{if: } \sin x + \cos x > 0 &\rightarrow \sqrt{2} \cos x = \sqrt{2} \sin x \rightarrow \sin x = \cos x \rightarrow x = \left\{ k\pi + \frac{\pi}{4} \right\} \\ \text{if: } \sin x + \cos x < 0 &\rightarrow \sqrt{2} \cos x = -\sqrt{2} \sin x \rightarrow \sin x = -\cos x \rightarrow x = \left\{ 2k\pi - \frac{\pi}{4} \right\} \end{aligned} \left. \vphantom{\begin{aligned} \text{if: } \sin x + \cos x > 0 \\ \text{if: } \sin x + \cos x < 0 \end{aligned}} \right\} \xrightarrow{n} x = \left\{ k\pi + \frac{\pi}{4} \right\}$$

$$\text{الف) } \sin^2 x - \cos^2 x = 1 \rightarrow \sin^2 x - \cos^2 x = 1 \rightarrow -\cos^2 x = 1 \rightarrow \cos^2 x = -1 \rightarrow \quad (10)$$

$$x = \{2k\pi + \pi\} \rightarrow x = \left\{ k\pi + \frac{\pi}{2} \right\} \rightarrow \text{بين اين ٢ بازيه ٢ جواب داريم.}$$

$$\begin{aligned} \text{ب) } \sin^2 x - \cos^2 x = -1 &\rightarrow \cos^2 x = 1, \sin^2 x = 0 \rightarrow x = 0, 2\pi \\ &\rightarrow \cos^2 x = 0, \sin^2 x = 1 \rightarrow x = \frac{3\pi}{2} \end{aligned} \left. \vphantom{\begin{aligned} \text{ب) } \sin^2 x - \cos^2 x = -1 \\ \rightarrow \cos^2 x = 0, \sin^2 x = 1 \end{aligned}} \right\} \rightarrow x = \left\{ 0, \frac{3\pi}{2}, 2\pi \right\}$$

٣ جواب داريم، اين بازيه داريم