

دوازدهم فصل A

الف) $\cos 2\alpha = 3 \sin \alpha - 1$

$1 + \cos 2\alpha = 3 \sin \alpha$

$2 \cos^2 \alpha = 3 \sin \alpha \Rightarrow 2(1 - \sin^2 \alpha) = 3 \sin \alpha \Rightarrow$

$2 - 2 \sin^2 \alpha = 3 \sin \alpha \Rightarrow 2 \sin^2 \alpha + 3 \sin \alpha - 2 = 0$

$\Delta = 9 + 16 = 25 \rightarrow \sin \alpha = \frac{-3 \pm 5}{4}$ $\frac{2}{4} = \frac{1}{2} \checkmark$
 $\frac{-1}{2} = -\frac{1}{2} \times$

$\sin \alpha = \frac{1}{2} \Rightarrow \begin{cases} \alpha = 2k\pi + \frac{\pi}{6} \\ \alpha = 2k\pi + \frac{5\pi}{6} \end{cases}$



ب) $\cos 2\alpha + \cos \alpha - 2 = 0$

$2 \cos^2 \alpha - 1 + \cos \alpha - 2 = 0 \Rightarrow 2 \cos^2 \alpha + \cos \alpha - 3 = 0$

$\Delta = 1 + 24 = 25 \Rightarrow \cos \alpha = \frac{-1 \pm 5}{4}$ $1 \checkmark$
 $-\frac{3}{2} = -\frac{3}{2} \times$

$\cos \alpha = 1 \Rightarrow \alpha = 2k\pi$



الف) $\sin^2 \alpha - \cos^2 \alpha = \sin \alpha$

$(\sin^2 \alpha - \cos^2 \alpha)(\sin^2 \alpha - \cos^2 \alpha) = \sin \alpha$

$-\cos 2\alpha = \sin \alpha$

$\sin(\frac{\pi}{2} + \alpha) = \sin \alpha$

$\rightarrow \alpha = 2k\pi + \frac{\pi}{2} + \alpha$

$\alpha = 2k\pi + \frac{\pi}{2} \Rightarrow \alpha = 2k\pi + \frac{\pi}{2}$

$\rightarrow \alpha = 2k\pi + \pi - \frac{\pi}{2} - \alpha$

$\alpha = 2k\pi + \frac{\pi}{2} \Rightarrow \alpha = 2k\pi + \frac{\pi}{2}$

$$\rightarrow r \cos^2 \alpha + r \sin \alpha \cos \alpha = 1 \quad | - r \cos^2 \alpha$$

$$\sin \alpha = - \cos \alpha$$

$$\sin \alpha = \sin \left(\frac{\pi}{r} + \alpha \right)$$

$$\rightarrow \alpha = K\pi + \frac{\pi}{r} + \alpha \quad \times$$

$$\rightarrow \alpha = K\pi + \pi - \frac{\pi}{r} - \alpha$$

$$\alpha = K\pi + \frac{\pi}{r} \rightarrow \alpha = \frac{K\pi}{r} + \frac{\pi}{r}$$

$$\text{ب) } \cot \alpha (r \cos \alpha + 1) = \frac{1}{\sin \alpha}$$

$$\times \sin \alpha$$

$$\cos \alpha (r \cos \alpha + 1) = 1 \Rightarrow r \cos^2 \alpha + \cos \alpha - 1 = 0$$

$$a+c=b \rightarrow \cos \alpha = -1 \quad \cos \alpha = \frac{1}{r}$$

$$\oplus \quad \alpha = K\pi + \pi \quad \times$$

$\sin \alpha = 0$ $\times$

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$$\oplus \quad \alpha = K\pi \pm \frac{\pi}{r}$$

$$\rightarrow r \sin^2 \alpha - \sin \alpha \cos \alpha + \cos^2 \alpha = r$$

$$r \sin^2 \alpha + \cos^2 \alpha - \sin \alpha \cos \alpha = r \div \sin \alpha$$

$$r + \cot^2 \alpha - \cot \alpha = r \left(\frac{1}{\sin^2 \alpha} \right) \rightarrow 1 + \cot^2 \alpha$$

$$\times \sin^2 \alpha \quad \cot^2 \alpha - \cot \alpha = r + r \cot^2 \alpha$$

$$\cot^2 \alpha + \cot \alpha = 0 \rightarrow \cot \alpha (\cot \alpha + 1) = 0$$

$$\rightarrow \cot \alpha = 0 \quad \oplus \rightarrow \alpha = K\pi + \frac{\pi}{r}$$

$$\rightarrow \cot \alpha = -1 \quad \oplus \rightarrow \alpha = K\pi - \frac{\pi}{r}$$

$$\text{ج) } \cos \left(\frac{\pi}{r} + \alpha \right) \cos \left(\alpha - \frac{\pi}{r} \right) = \frac{1}{r}$$

$$\cos \left(\alpha - \frac{\pi}{r} \right) = \sin \left(\frac{\pi}{r} + \left(\alpha - \frac{\pi}{r} \right) \right) = \sin \left(\frac{\pi}{r} + \alpha \right)$$

$$\cos \left(\frac{\pi}{r} + \alpha \right) \sin \left(\frac{\pi}{r} + \alpha \right) = \frac{1}{r}$$

$$\frac{1}{r} \sin \left(\alpha + \frac{\pi}{r} \right) = \frac{1}{r} \Rightarrow \sin \left(\alpha + \frac{\pi}{r} \right) = \frac{1}{r} \rightarrow \sin \left(\alpha + \frac{\pi}{r} \right) = \sin \frac{\pi}{r}$$

المطلوب

$$\sin\left(r\alpha + \frac{\pi}{r}\right) = \sin\frac{\pi}{r}$$

$$\rightarrow r\alpha + \frac{\pi}{r} = rK\pi + \frac{\pi}{r} \rightarrow r\alpha = rK\pi - \frac{r\pi}{r} \Rightarrow \alpha = K\pi - \frac{\pi}{r}$$

$$\rightarrow r\alpha + \frac{\pi}{r} = rK\pi + \frac{2\pi}{r} \rightarrow r\alpha = rK\pi + \frac{r\pi}{r} \Rightarrow \alpha = K\pi + \frac{\pi}{r}$$

$$\rightarrow \cos\left(r\alpha - \frac{\pi}{r}\right) = -\sin r\alpha \rightarrow \sin(-r\alpha)$$

$$\cos\left(r\alpha - \frac{\pi}{r}\right) = \sin\left(\frac{\pi}{r} - \left(r\alpha - \frac{\pi}{r}\right)\right)$$

$$\sin\left(\frac{\pi}{r} - \left(r\alpha - \frac{\pi}{r}\right)\right) = \sin(-r\alpha)$$

$$\rightarrow -r\alpha = rK\pi + \frac{\pi}{r} - r\alpha + \frac{\pi}{r} \quad *$$

$$\rightarrow -r\alpha = rK\pi + \pi - \frac{\pi}{r} + r\alpha - \frac{\pi}{r}$$

$$-r\alpha = rK\pi + \frac{\pi}{r} - \frac{\pi}{r}$$

$$-r\alpha = rK\pi + \frac{\pi}{r} - \frac{\pi}{r} \Rightarrow \alpha = \frac{K\pi}{r} - \frac{\pi}{r}$$

$$r\sin r\alpha \cos r\alpha$$

$$\frac{\sin r\alpha + \sin r\alpha}{\sin r\alpha} - \sin r\alpha = \cos r\alpha \quad -a$$

$$r\cos r\alpha + 1 = \cos r\alpha + \sin r\alpha \Rightarrow r\cos r\alpha = 0 \quad \oplus$$

$$\Rightarrow r\alpha = K\pi + \frac{\pi}{r} \Rightarrow \alpha = \frac{K\pi}{r} + \frac{\pi}{r} \quad \oplus$$

$$* \sin r\alpha \neq 0 \quad \text{باز هم به همین ترتیب این کار را تکرار می‌کنیم}$$

$$\alpha \in [-\pi, \frac{\pi}{2}] \quad -4$$

$$\sin \frac{\alpha}{r}$$

$$\frac{1}{1 + \cos \frac{\alpha}{r}} = \frac{1}{\sin \frac{\alpha}{r}} + \cot \frac{\alpha}{r}$$

$$\cancel{r \sin \frac{\alpha}{r}} \frac{\cancel{\cos \frac{\alpha}{r}}}{\cancel{r \cos \frac{\alpha}{r}}} =$$

$$\frac{1 + \cos \frac{\alpha}{r}}{\sin \frac{\alpha}{r}}$$

$$\rightarrow r \cos \frac{\alpha}{r}$$

$$\cancel{r \cos \frac{\alpha}{r}}$$

$$\frac{\sin \frac{\alpha}{r}}{\sin \frac{\alpha}{r}}$$

$$\rightarrow r \sin \frac{\alpha}{r} \cancel{\cos \frac{\alpha}{r}}$$

$$\tan \frac{\alpha}{r} = \cot \frac{\alpha}{r} \Rightarrow \tan \frac{\alpha}{r} = \tan \left(\frac{\pi}{r} - \frac{\alpha}{r} \right)$$

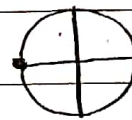
$$\frac{\alpha}{r} = k\pi + \frac{\pi}{r} - \frac{\alpha}{r} \Rightarrow \frac{\alpha}{r} = k\pi + \frac{\pi}{r} \Rightarrow \alpha = r k \pi + \pi$$

$$\alpha = \pi$$

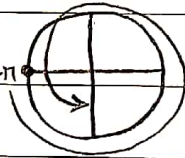
$$1 + \cos \frac{\pi}{r} \neq 0 \checkmark$$

$$\sin \frac{\pi}{r} \neq 0 \checkmark$$

$$\sin \frac{\pi}{r} \neq 0 \checkmark$$



$$\alpha \in [-\pi, \frac{\pi}{2}]$$



قدرة على تقاطع
جواب

$$|-\pi - (\pi)| = 2\pi$$

$$\cos^2 \alpha - \sin^2 \alpha \cos^2 \alpha = 1$$

$$\alpha \in [0, 2\pi] \quad -5$$

$$\cos^2 \alpha - 1 = \sin^2 \alpha \cos^2 \alpha$$

$$-\sin^2 \alpha = \sin^2 \alpha \cos^2 \alpha$$

$$\sin^2 \alpha \cos^2 \alpha + \sin^2 \alpha = 0$$

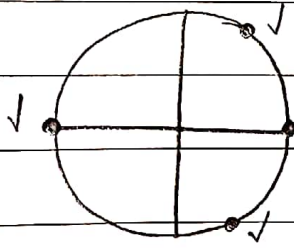
$$\sin^2 \alpha (\cos^2 \alpha + 1) = 0 \rightarrow$$



$$\sin \alpha = 0 \Rightarrow \alpha = k\pi$$

$$\cos^2 \alpha = -1 \Rightarrow \alpha = k\pi + \pi$$

$$\alpha = \frac{r k \pi}{r} + \frac{\pi}{r}$$



$$\Rightarrow \text{جواب: } \frac{\pi}{2}$$

$$\frac{\sin \alpha}{\cos \alpha}$$

$$1 - \tan \alpha$$

$$= \tan^2 \alpha$$

$$\Rightarrow \frac{\cos \alpha - \sin \alpha}{\cos \alpha + \sin \alpha} = \frac{\sin^2 \alpha}{\cos^2 \alpha}$$

$$\frac{\sin \alpha}{\cos \alpha}$$

$$\cos^2 \alpha \cos \alpha - \sin \alpha \cos^2 \alpha = \cos \alpha \sin^2 \alpha + \sin \alpha \sin^2 \alpha$$

$$\cos^2 \alpha \cos \alpha - \sin^2 \alpha \sin \alpha = \cos \alpha \sin^2 \alpha + \sin \alpha \cos^2 \alpha$$

$$\cos^2 \alpha = \sin^2 \alpha \Rightarrow \sin \left(\frac{\pi}{2} - \alpha \right) = \sin \alpha$$

$$\alpha = r k \pi + \frac{\pi}{r} - \alpha$$

$$\alpha = r k \pi + \pi - \frac{\pi}{r} + \alpha \cdot X$$

$$\alpha = r k \pi + \frac{\pi}{r} \rightarrow \alpha = \frac{k \pi}{r} + \frac{\pi}{r}$$

$$\sqrt{1 + \sin 2\alpha} = \sqrt{2} \cos \alpha$$

$$\alpha \in [0, \pi] \quad \alpha$$

$$\sqrt{(\sin \alpha + \cos \alpha)^2} = \sqrt{2} \cos \alpha$$

$$|\sin \alpha + \cos \alpha| = \sqrt{2} \cos \alpha$$

$$\sqrt{2} \sin \left(\alpha + \frac{\pi}{4} \right) = \sqrt{2} \cos \alpha$$

$$\sin \left(\alpha + \frac{\pi}{4} \right) = \sin \left(\frac{\pi}{4} - \alpha \right)$$

$$\alpha + \frac{\pi}{4} = 2K\pi + \frac{\pi}{4} - \alpha \quad \Leftrightarrow \quad \alpha + \frac{\pi}{4} = 2K\pi + \pi - \frac{\pi}{4} + \alpha$$

$$2\alpha = 2K\pi + \frac{\pi}{4} - \frac{\pi}{4}$$

$$-\alpha = 2K\pi + \pi - \frac{\pi}{4} - \frac{\pi}{4}$$

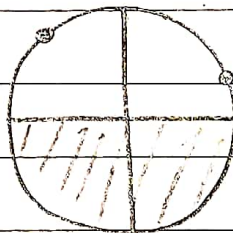
$$2\alpha = 2K\pi + \frac{\pi}{2}$$

$$-\alpha = 2K\pi + \pi - \frac{\pi}{2}$$

$$\alpha = \frac{2K\pi}{2} + \frac{\pi}{4}$$

$$-\alpha = 2K\pi + \frac{\pi}{2}$$

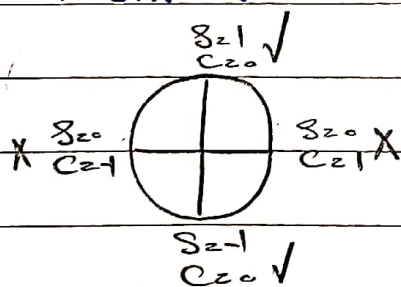
$$\alpha = \frac{2K\pi}{2} - \frac{\pi}{4}$$



رد جواب

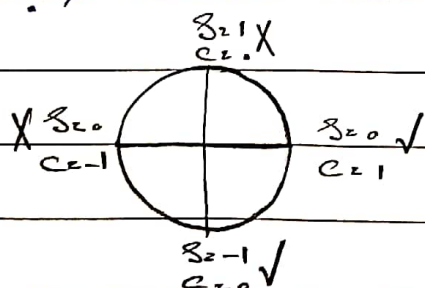
$$\sin^4 \alpha - \cos^4 \alpha = 1 \quad \text{الف)}$$

$$\alpha \in [0, \pi] \quad 1.$$



$$\Rightarrow \alpha = \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$$

$$\sin^4 \alpha - \cos^4 \alpha = -1 \quad \text{ب)}$$



$$\Rightarrow \alpha = \left\{ 0, \frac{3\pi}{4}, \pi \right\}$$