سارا امینی راد **۱۶**

۱) الف) **۱/۵**

$\cos 2x = 3\sin x - 1 \rightarrow \cos 2x = 1 - 2\sin^2 x$

$\rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow -2\sin^2 x - 3\sin x + 2 = 0 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\rightarrow (2\sin x - 1)(\sin x + 2) = 0 \rightarrow \sin x = \frac{1}{2} \rightarrow x = 2k\pi + \frac{5\pi}{6}$ ✓

$\frac{1}{2}$, -2 ✗ $\qquad x = 2k\pi + \frac{\pi}{6}$ ✓

ب)

دقت کن!

$\cos 2x + \cos x - 2 = 0 \rightarrow \cos 2x = 2\cos^2 x - 1 \rightarrow \underbrace{2\cos^2 x + \cos x - 3 = 0}$

$(2\cos x + 3)(\cos x - 1) \rightarrow \cos x = 1 \rightarrow x = 2k\pi$

$\cos x = -\frac{3}{2}$ غ ق ق

$\rightarrow (2\cos x + 3)(\cos x + 1) = 0$

$-\frac{3}{2}$ ✗ $\qquad -1 \rightarrow x = 2k\pi + \pi$

۲) الف) **۲**

$\sin^4 x - \cos^4 x = \sin 4x \rightarrow (\sin^2 x - \cos^2 x)\underbrace{(\sin^2 x + \cos^2 x)}_{1}$

$\sin^2 x - \cos^2 x = -\cos 2x$

$\sin 4x = 2\sin 2x \cos 2x \longrightarrow -\cos 2x = 2\sin 2x \cos 2x \rightarrow \cos 2x = 0$

$2x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

$-1 = 2\sin 2x \rightarrow x = k\pi + \frac{7\pi}{12}$ ✓

$x = k\pi - \frac{\pi}{12}$ ✓

ب)

$2\cos^2 x + 2\sin x \cos x = 1 \longrightarrow 2\sin x \cos x = \sin 2x$

$1 + \cos 2x + \sin 2x = 1 \longrightarrow \cos 2x + \sin 2x = 0 \rightarrow \sin 2x = -\cos 2x$

$\tan 2x = -1 \longrightarrow 2x = -\frac{\pi}{4} + k\pi \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$ ✓

۳) الف) **۱/۷۵**

$\sin x \neq 0$

$\times \sin x$

$\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \longrightarrow \cos x (2\cos x + 1) = 1 \rightarrow 2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$-1 \rightarrow x = 2k\pi + \pi$ ✗

اگر $\cos x = -1$ باشد $\sin x = 0$ است و عبارت اصلی تعریف نشده می‌شود!

$\frac{1}{2}$ $\qquad x = 2k\pi + \frac{\pi}{3}$ ✓

ب)

$2\sin^2 x - \sin x \cos x + \cos^2 x = 2 \xrightarrow{\div \sin^2 x} 2 - \cot x + \cot^2 x = 2(1 + \cot^2 x)$

$\rightarrow 2 - \cot x + \cot^2 x = 2 + 2\cot^2 x \rightarrow \cot^2 x + \cot x = 0 \rightarrow \cot x(\cot x + 1) = 0$

0: $x = k\pi + \frac{\pi}{2}$ $\qquad -1 \rightarrow x = k\pi - \frac{\pi}{4}$ ✓

(سارا امینی راد)

۴) الف)

$\cos(\frac{\pi}{6}+x)\cos(x-\frac{\pi}{6})=\frac{1}{4} \to \frac{1}{2}[\cos(A+B)+\cos(A-B)]$

$\hookrightarrow A+B=(\frac{\pi}{6}+x)+(x-\frac{\pi}{6})=2x \to A-B=\frac{\pi}{6}+x-(x-\frac{\pi}{6})=\frac{\pi}{3}$

$\cos A\cos B=\frac{1}{2}(\cos 2x+\cos\frac{\pi}{3}) \longrightarrow \cos\frac{\pi}{2}=0$

$\frac{1}{2}\cos 2x=\frac{1}{4} \to \cos 2x=\frac{1}{2} \to 2x=2k\pi+\frac{\pi}{3} \to x=k\pi+\frac{\pi}{6}$

$2x=2k\pi+\frac{5\pi}{3} \to x=k\pi+\frac{5\pi}{6}$

۲

ب)

$\cos(2x-\frac{\pi}{9})=-\sin 2x \longrightarrow -\sin 2x=\cos(2x+\frac{\pi}{2}) \to \cos(2x-\frac{\pi}{9})=\cos(2x+\frac{\pi}{2})$

$\to 2x-\frac{\pi}{9}=\cancel{2x}+\frac{\pi}{2}+2k\pi \to \cancel{2k\pi} \quad 2x-\frac{\pi}{9}=-2x-\frac{\pi}{2}+2k\pi$

$\to 4x=-\frac{\pi}{2}+\frac{\pi}{9}+2k\pi \longrightarrow 4x=-\frac{7\pi}{18}+2k\pi \to x=-\frac{7\pi}{72}+\frac{k\pi}{2}$

۵)

$\frac{\sin 4x+\sin 2x}{\sin 3x}=1 \quad \sin 4x+\sin 2x=\sin 3x \to \sin(2x)+\sin(4x)=2\sin(3x)\cos(x)$

$2\sin(3x)\cos(x)=\sin(3x) \to \sin 3x=0 \to 3x=k\pi \to x=\frac{k\pi}{3}$

$\hookrightarrow 2\cos(x)-1=0 \to \cos(x)=\frac{1}{2} \to x=\pm\frac{\pi}{3}+2k\pi$

۱,۲۵

۶)

$\tan\frac{x}{4}=\cot\frac{x}{2}+\cot\frac{x}{2} \to \tan\frac{x}{4}=\cot\frac{\pi}{4} \to \tan(\frac{x}{4})=1 \to x=\pi+2k\pi$

$|\pi-(-\pi)|=2\pi$

۲

۷)

۰

۸)

$\tan(\frac{\pi}{4}-x)=\frac{1-\tan x}{1+\tan x} \to \frac{\pi}{4}-x=3x+k\pi \to 4x=\frac{\pi}{4}+k\pi \to x=\frac{k\pi}{4}+\frac{\pi}{16}$

۲

(سارا امینی‌زاد)

۹) ۱/۵

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \longrightarrow 1+\sin 2x = 2\cos^2 2x = 1+\cos 4x$

$\sin 2x = \cos 4x = \sin(\frac{\pi}{2} - 4x) \longrightarrow 2x = \frac{\pi}{2} - 4x + 2k\pi \to 6x = \frac{\pi}{2} + 2k\pi$ $\hookrightarrow x = \frac{k\pi}{3} + \frac{\pi}{12}$

$k=0 \to x = \frac{\pi}{12}$ $\quad k=2 \to x = \frac{3\pi}{4}$ ؟ در بازه‌ی $[0, \pi]$

$k=1 \to x = \frac{5\pi}{12}$ × به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود چون جواب رادیکال است غ ق ق است

۱۰) $\sin^4 x - \cos^4 x = 1 \to \underbrace{(\sin^2 x - \cos^2 x)}(\sin^2 x + \cos^2 x) = 1$

الف) $-\cos 2x = 1 \to \cos 2x = -1 \longrightarrow 2x = 2k\pi + \pi \longrightarrow x = k\pi + \frac{\pi}{2} \sim x = \frac{\pi}{2}, \frac{3\pi}{2}$

۲

ب) $(\sin x - \cos x)(\sin^2 x + \sin x\cos x + \cos^2 x) =$

$\sin x = 0 \quad \cos x = 1$

$\cos x = 0 \quad \sin x = -1$

$x = 0$

$x = \frac{3\pi}{2}$

$x = 2\pi$

سوال ۵

$$\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x = 1$$

$$\frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \to \cos 2x = 0$$

$$2x = k\pi + \frac{\pi}{2} \to \boxed{x = k\pi + \frac{\pi}{4}}$$

سوال ۷

$$\cos^2 x - \sin^2 x\cos 3x = 1 \xrightarrow{\cos^2 x = 1 - \sin^2 x}$$

$$\cancel{1} - \sin^2 x - \sin^2 x\cos 3x = \cancel{1} \longrightarrow \sin^2 x(1 + \cos 3x) = 0$$

$\hookrightarrow \sin x = 0$

$\hookrightarrow \cos 3x = -1$

$\sin x = 0 \longrightarrow x = k\pi \longrightarrow x = 0, \pi, 2\pi$ (۳) (۲) (۱)

$\cos 3x = -1 \to 3x = 2k\pi + \pi \to x = \frac{2k\pi + \pi}{3} \to x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ (۵) (۴)

معادله ۵ جواب دارد