۱۹٫۷۵ آفرین خیلی عالی نوشتی!

نام و نام خانوادگی ....آنیتا برین.................. پاسخنامه تشریحی تکلیف شماره ..۱۶.... کلاس ..ریاضی دهم A......

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الف) $\cos 2x = 3\sin x - 1 \rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \rightarrow \sin x = -2$ ✗

$\searrow \sin x = \frac{1}{2}$ ✓

$\boxed{x = 2k\pi + \frac{\pi}{6}}$ یا $\boxed{x = 2k\pi + \frac{5\pi}{6}}$

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0 \rightarrow \cos x = -\frac{3}{2}$ ✗

$\searrow \cos x = 1$ ✓

$\boxed{x = 2k\pi}$

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الف) $\sin^4 x - \cos^4 x = \sin 4x \rightarrow \sin^2 x - \cos^2 x = \sin 4x \rightarrow -\cos 2x = 2\sin 2x \cos 2x$

$\rightarrow \cos 2x = 0 \rightarrow 2x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\rightarrow \sin 2x = -\frac{1}{2} \rightarrow 2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12}$

$\rightarrow 2x = 2k\pi - \frac{5\pi}{6} \rightarrow x = k\pi - \frac{5\pi}{12}$

ب) $2\cos^2 x + 2\sin x \cos x = 1 \rightarrow \cancel{1} + \cos 2x + \sin 2x = \cancel{1} \rightarrow \sin 2x + \cos 2x = 0 \rightarrow 2x = k\pi + \frac{\pi}{4}$

$x = \frac{k\pi}{2} + \frac{\pi}{8}$

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الف) $\frac{\cos x}{\cancel{\sin x}}(2\cos x + 1) = \frac{1}{\cancel{\sin x}} \xrightarrow{\sin x \neq 0} 2\cos^2 x + \cos x - 1 = 0 \rightarrow \cos x = -1$

$\searrow \cos x = \frac{1}{2}$ $+ \sin x \neq 0 \Rightarrow$

$x = 2k\pi \pm \frac{\pi}{3}$

ب) $\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} - \sin x \cos x = 2 \rightarrow \sin^2 x - \sin x \cos x = 1 \rightarrow \cos x = 0 \rightarrow x = k\pi + \frac{\pi}{2}$

$\div \cos^2 x$: $\cancel{\tan^2 x} - \tan x = \cancel{\tan^2 x} + 1$

$\tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}$

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الف) $\cos(\frac{\pi}{4} + x)\underbrace{\cos(x - \frac{\pi}{4})}_{\cos(\frac{\pi}{4} - x) = \sin(\frac{\pi}{4} + x)} = \frac{1}{4} \rightarrow \sin(x + \frac{\pi}{4})\cos(x + \frac{\pi}{4}) = \frac{1}{4} \xrightarrow{\times 2} \underbrace{\sin(2x + \frac{\pi}{2})}_{\cos 2x} = \frac{1}{2}$

$\cos 2x = \frac{1}{2}$

$\Rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = \underbrace{-\sin 2x}_{\cos(\frac{\pi}{2} + 2x)} \rightarrow \cancel{2x} - \frac{\pi}{9} = \cancel{2x} + \frac{\pi}{2} + 2k\pi$ ✗

$\searrow 2x - \frac{\pi}{9} = -2x - \frac{\pi}{2} + 2k\pi \rightarrow x = \frac{k\pi}{2} - \frac{7\pi}{72}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \rightarrow \frac{\sin 4x + \sin 2x}{\sin 2x} = \cancel{\sin^2 x + \cos^2 x}^{\,1}$ $\sin 2x \neq 0$

$2x \neq k\pi$

$\sin 4x + \cancel{\sin 2x} = \cancel{\sin 2x} \rightarrow \sin 4x = 0 \rightarrow 4x = k\pi \rightarrow x = \frac{k\pi}{4}$ I $\qquad x \neq \frac{k\pi}{2}$ II

$\mathrm{I}, \mathrm{II} \rightarrow x = k\pi \pm \frac{\pi}{4}$

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Scanned with CamScanner

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$\alpha = \frac{x}{2} \rightarrow \frac{\sin\alpha}{1+\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$ $\quad \sin\alpha \neq 0$, $\cos\alpha \neq -1$

$\sin^2\alpha = 1+\cos^2\alpha + 2\cos\alpha \rightarrow \cancel{\sin^2\alpha} = \cancel{\sin^2\alpha} + \cos^2\alpha + \cos^2\alpha + 2\cos\alpha$

$2\cos^2\alpha + 2\cos\alpha = 0 \rightarrow 2\cos\alpha(\cos\alpha + 1) = 0 \rightarrow \cos\alpha = 0 \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2}$ $\quad x = 2k\pi + \pi$

$\rightarrow \cos\alpha = -1$ ✗

در بازه $[-\pi, \frac{3\pi}{2}] \rightarrow -\pi, \pi$ جواب‌ها $\rightarrow |-\pi - \pi| = 2\pi$

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$\cos^2x - \sin^2x\cos 3x = 1 \rightarrow \underbrace{\cos^2x - 1}_{-\sin^2x} = \sin^2x\cos 3x \rightarrow \sin x = 0 \rightarrow x = k\pi$

$\rightarrow \cos 3x = -1 \rightarrow 3x = 2k\pi + \pi \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$

در بازه $[0, 2\pi] \rightarrow$ ۵ جواب

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$\frac{1-\tan x}{1+\tan x} = \tan 3x \rightarrow \tan(\frac{\pi}{4} - x) = \tan 3x \rightarrow 3x = \frac{\pi}{4} - x + k\pi \rightarrow 4x = k\pi + \frac{\pi}{4}$

$x = \frac{k\pi}{4} + \frac{\pi}{16}$

$\tan x \neq -1 \rightarrow x \neq k\pi + \frac{\pi}{4}$

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$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \rightarrow \sqrt{(\sin x + \cos x)^2} = \sqrt{2}(\cos^2x - \sin^2x)$

$|\sin x + \cos x| = \sqrt{2}(\sin x + \cos x)(\cos x - \sin x) \rightarrow \cos x - \sin x = \pm\frac{\sqrt{2}}{2} \rightarrow \sin(x - \frac{\pi}{4}) = \pm\frac{\sqrt{2}}{2}$

$x - \frac{\pi}{4} = k\pi \pm \frac{\pi}{4} \rightarrow x = k\pi$; $x = k\pi + \frac{\pi}{2}$ $\Big\} \xrightarrow{[0,\pi]}$ سه جواب

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به ازای $x = \frac{5\pi}{4}$ $\cos 2x$ منفی می‌شود، یک جواب را اشتباه [illegible] است

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الف) $\sin^4x - \cos^4x = 1 \rightarrow \cancel{(\sin^2x + \cos^2x)}(\sin^2x - \cos^2x) = 1 \rightarrow \cos 2x = -1$ $\quad 2x = 2k\pi + \pi$ $\quad x = k\pi + \frac{\pi}{2}$

در بازه $[0, 2\pi] \rightarrow$ ۲ جواب $\rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

ب) $\sin^3x - \cos^3x = -1 \rightarrow (\sin x - \cos x)(1 + \sin x\cos x) = -1 \rightarrow \sin x\cos x = 0$

در بازه $[0, 2\pi] \rightarrow$ ۳ جواب $\rightarrow 0, \frac{3\pi}{2}, 2\pi$

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سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2 - 2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0 \to \sin 2x = -1$$

یا $\sin 2x = +\frac{1}{2}$

$$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12} \checkmark \text{ یا } \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ ق ق است

* معادله ۲ جواب دارد