تصحیح کل سرخی - تکلیف شماره ۱۴ دوازدهم دفتر A

$-2\sin^2 x+1$

① $\cos 2x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \rightarrow \sin^2 x + 3\sin x - 4 = 0 \rightarrow (\sin x + 4)(\sin x - 1) = 0$

$\sin x = -2$ ✗✗ غ ق ق

$\sin x = \frac{1}{2}$ ✓✓ $\rightarrow \sin x = \sin\frac{\pi}{6}$

① $x = 2k\pi + \frac{\pi}{6}$

② $x = 2k\pi + \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

② $\underbrace{\sin^4 x - \cos^4 x}_{\sin^2 x - \cos^2 x} = \sin 4x \rightarrow \underbrace{-\cos 2\alpha}_{\sin(2\alpha - \frac{\pi}{2})} = \sin 4\alpha$

① $2\alpha - \frac{\pi}{2} = 2k\pi + 4\alpha \rightarrow -2\alpha = 2k\pi + \frac{\pi}{2}$

$\rightarrow \alpha = -k\pi - \frac{\pi}{4}$

② $2\alpha - \frac{\pi}{2} = 2k\pi + \pi - 4\alpha \rightarrow 6\alpha = 2k\pi + \frac{3\pi}{2}$

$\rightarrow \alpha = \frac{k\pi}{3} + \frac{\pi}{4}$

③ الف) $\frac{\cos\alpha}{\sin\alpha}(2\cos\alpha + 1) = \frac{1}{\cancel{\sin\alpha}} \rightarrow 2\cos^2\alpha + \cos\alpha - 1 = 0$

$\rightarrow \cos\alpha = -1$ غ ق ق

$\rightarrow \cos\alpha = \frac{1}{2} \rightarrow \alpha = 2k\pi \pm \frac{\pi}{6}$

ب) $2\sin^2 x - \sin x\cos x + \underbrace{\cos^2 x}_{1-\sin^2} = 2 \rightarrow \sin^2 - \sin\cos = 1 \rightarrow \underbrace{\sin^2 - 1}_{-\cos} = \sin\cancel{\cos} \rightarrow \sin x = -1$ $\frac{3\pi}{2}$

$x = k\pi \pm \frac{\pi}{2}$

④ الف) $\cos(\frac{\pi}{4} + x)\underbrace{\cos(x - \frac{\pi}{4})}_{\sin(\frac{\pi}{4}+x)} = \frac{1}{4} \rightarrow \frac{1}{\cancel{2}}\sin(\frac{\pi}{2} + 2x) = \frac{\cancel{1}}{\cancel{4}}\frac{1}{2} \rightarrow \cos 2x = \frac{1}{2}$

$\cos 2x = \cos\frac{\pi}{3} \rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = \underbrace{-\sin 2x}_{\cos(\frac{\pi}{2} + 2x)}$

① $\cancel{2x} - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + \cancel{2x}$ غ ق ق

② $2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \rightarrow 4x = 2k\pi - \frac{7\pi}{18} \rightarrow x = \frac{k\pi}{2} - \frac{7\pi}{72}$

⑤ $\frac{\sin 4x + \sin 2x}{\sin 2x}$ $-\sin^2 = \cos^2 \rightarrow \sin^2 + \cos^2 = 1 \rightarrow \sin 4x + \cancel{\sin 2x} = \cancel{\sin 2x} \rightarrow \sin 4x = 0$

$\sin 4x = \sin\pi$

① $4x = 2k\pi + \pi \rightarrow \frac{k\pi}{2} + \frac{\pi}{4}$

② $4x = 2k\pi + \cancel{\pi - \pi} \rightarrow x = \frac{k\pi}{2}$ غ ق ق

(۶) $\dfrac{\cancel{\sin x/۲}}{1+\cancel{\cos x/۲}} = \dfrac{1}{\sin x/۲} + \cot x/۲ \rightarrow \dfrac{\sin x/۲}{1+\cos x/۲} = \dfrac{1+\cos x/۲}{\sin x/۲} \rightarrow \cancel{\sin^۲ x/۲} = \cos^۲ x/۲ + ۲\cos x/۲ + \underbrace{1}_{\cos^۲ x/۲ + \cancel{\sin^۲ x/۲}}$

$۲\cos^۲ x/۲ + ۲\cos x/۲ = 0 \rightarrow ۲\cos x/۲ (\cos x/۲ + 1) = 0$

$\rightarrow \cos x/۲ = -1$ ق ق غ

$\rightarrow ۲\cos x/۲ = 0 \longrightarrow \cos x/۲ = \cos \frac{\pi}{۲} \rightarrow \frac{x}{۲} = ۲k\pi \pm \frac{\pi}{۲} \rightarrow x = ۴k\pi \pm \pi$

جوابها: $\{0, \quad \}$

(۷) $\cos^۶ x - \sin^۲ x \cos^۲ x = 1 \rightarrow \cos^۶ x + \sin^۲ x \rightarrow \sin^۲ x (1+\cos ۲x) = 0$ ① $\sin^۲ x = 0 \rightarrow \sin x = 0 = \sin \pi$ ② $\cos ۲x = -1 = \cos(\pi)$

$\xrightarrow{①} x = ۲k\pi + \pi \;,\; x = ۲k\pi + \cancel{\pi}$

$\xrightarrow{②} ۲x = ۲k\pi \pm \pi \rightarrow \frac{۲k\pi}{۲} \pm \frac{\pi}{۲} = x$

(۸) $\dfrac{1-\tan x}{1+\tan x} = \tan ۳x \rightarrow \dfrac{\dfrac{\cos x - \sin x}{\cancel{\cos x}}}{\dfrac{\cos x + \sin x}{\cancel{\cos x}}} = \dfrac{\sin ۳x}{\cos ۳x} \Rightarrow \cos ۳x \cos x - \cos ۳x \sin x =$
$\sin ۳x \cos x + \sin ۳x \sin x$

$\longrightarrow \underbrace{\cos ۳x \cos x - \sin ۳x \sin x}_{\cos ۴x} = \underbrace{\cos ۳x \sin x + \sin ۳x \cos x}_{\sin ۴x = \cos(۴x - \frac{\pi}{۲})} \Rightarrow ۴x = ۲k\pi \pm (۴x - \frac{\pi}{۲})$ ق ق غ

$۴x = ۲k\pi - ۴x + \frac{\pi}{۲} \rightarrow ۸x = ۲k\pi + \frac{\pi}{۲} \rightarrow x = \frac{k\pi}{۴} + \frac{\pi}{۱۶}$

(۹) $\sqrt{\underbrace{1+\sin ۲x}_{(\cos+\sin)^۲}} = \sqrt{۲}\cos ۲x \rightarrow \cos x + \sin x = \sqrt{۲}\cos^۲ x - \sqrt{۲}\sin^۲ x$

$\rightarrow \sin x(\sqrt{۲}\sin x + 1) = \cos x(\sqrt{۲}\cos x - 1)$

جوابها: $\left\{\frac{۳\pi}{۲}, \frac{\pi}{۲}\right\}$

(۱۰) الف) $\sin^۴ x - \cos^۴ x = 1 \longrightarrow \sin^۲ x - \cos^۲ x = 1 \rightarrow -\cos ۲x = 1 \rightarrow \cos ۲x = -1 = \cos\pi \rightarrow ۲x = ۲k\pi \pm \pi \rightarrow x = k\pi \pm \frac{\pi}{۲}$

ب) $\sin^۳ x - \cos^۳ x = -1 \xrightarrow{\text{حالت ها}} ① \cos^۳ x = 1 \rightarrow \cos x = 1 \rightarrow x = ۲k\pi \pm \pi \rightarrow$

$\rightarrow ② \sin^۳ x = -1 \rightarrow \sin x = -1 \rightarrow x = k\pi + \frac{۳\pi}{۲}$ یا $k\pi - \frac{\pi}{۲}$

جوابها: $\left\{\frac{۳\pi}{۲}, \ldots, \pi\right\}$