السیا خان کمری

۱۹ آفرین

۱) الف) $\cos 2\alpha = 3\sin\alpha - 1 \Rightarrow 2\sin^2\alpha + 3\sin\alpha - 2 = 0$ (۲)

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$2\sin^2\alpha + 3\sin\alpha - 4 = 0$

$(\sin\alpha + 4)(\sin\alpha - 1) = 0$

$\sin = -2$ ✗ ، $\sin = 1/2$ ✓

$\sin = 1/2 \Rightarrow 2k\pi + \frac{\pi}{6}$ و $2k\pi + \frac{5\pi}{6}$

ب) $\cos 2\alpha + \cos\alpha - 2 = 0 \Rightarrow 2\cos^2\alpha + \cos\alpha - 3 = 0$

$\cos 2\alpha = 2\cos^2\alpha - 1$

((جمع ضرایب = ۰))

$\cos\alpha = -\frac{3}{2}$ ✗

$\cos\alpha = 1$ ✓

$\cos\alpha = 1 \Rightarrow 2k\pi$

۲) الف) $\sin^4\alpha - \cos^4\alpha = \sin 4\alpha$ (۲)

$(\sin^2\alpha + \cos^2\alpha)(\sin^2\alpha - \cos^2\alpha)$

$(\sin^2\alpha + \cos^2\alpha) = 1$ ، $(\sin^2\alpha - \cos^2\alpha) = -\cos 2\alpha$

$\cos(2\alpha) = \sin(-4\alpha)$

$\cos 2\alpha = \cos(\frac{\pi}{2} + 4\alpha)$

① $2\alpha = 2k\pi + \frac{\pi}{2} + 4\alpha$

$2\alpha = -2k\pi - \frac{\pi}{2}$

$\alpha = \frac{-2k\pi - \frac{\pi}{2}}{2}$

② $2\alpha = 2k\pi - \frac{\pi}{2} - 4\alpha$

$6\alpha = 2k\pi - \frac{\pi}{2}$

$\alpha = \frac{2k\pi - \frac{\pi}{2}}{6}$

ب) $2\cos^2\alpha - 1 = -\sin 2\alpha \Rightarrow \cos 2\alpha = \cos(\frac{\pi}{2} + 2\alpha)$

$2\cos^2\alpha - 1 = \cos 2\alpha$

① $2\alpha = 2k\pi + \frac{\pi}{2} + 2\alpha$ ✗

② $2\alpha = 2k\pi - \frac{\pi}{2} - 2\alpha$

$4\alpha = 2k\pi - \frac{\pi}{2}$

$\alpha = \frac{2k\pi - \pi/2}{4}$

۳) الف) $\cot(2\cos\alpha + 1) = \frac{1}{\sin\alpha}$ (۲)

$\cot = \frac{\cos\alpha}{\sin\alpha}$

$2\cos^2\alpha + \cos\alpha - 1 = 0 \rightarrow \cos = -1$

« جمع ضرایب طرفین = وسطی » $\rightarrow \cos = 1/2$

① $\cos\alpha = -1 \Rightarrow \alpha = 2k\pi + \pi$

② $\cos\alpha = 1/2 \Rightarrow \alpha = 2k\pi \pm \frac{\pi}{3}$

★ شرط داشتیم $\sin\alpha \neq 0$ ، پس این جواب غ ق ق

s.a.m

الیاس خانی ترسی

ب) $2\sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha = 2\sin^2\alpha + 2\cos^2\alpha$

$\cos^2\alpha + \sin\alpha\cos\alpha = 0 \qquad \cos\alpha(\cos\alpha + \sin\alpha) = 0$

① $\cos\alpha = 0 \quad \alpha = k\pi + \frac{\pi}{2}$ ✓ ② $\cos\alpha = -\sin\alpha \rightarrow \alpha = k\pi + \frac{3\pi}{4}$ ✓

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④ 1/75

الف) $\cos(\frac{\pi}{4} + \alpha) \cdot \cos(\alpha - \frac{\pi}{4}) = \frac{1}{4}$

$\cos(\frac{\pi}{4} - \alpha)$ + قبلی

$\cos(\frac{\pi}{4} + \alpha) \times \sin(\frac{\pi}{4} + \alpha) = \frac{1}{4}$

شرط تشابه ... یک سینوس رو دیگه ... سینوس... میکنیم

$\cos\alpha = \cos\alpha$

$\frac{1}{2}\sin(\frac{\pi}{2} + 2\alpha) = \frac{1}{4}$

$\sin(\frac{\pi}{2} + 2\alpha) = \frac{1}{2} \qquad \cos 2\alpha = +\frac{1}{2}$

$\cos(2\alpha)$

~~$\alpha = \frac{2k\pi + \frac{2\pi}{3}}{2}$ ، $\alpha = \frac{2k\pi + \frac{4\pi}{3}}{2}$~~

$2\alpha = 2k\pi \pm \frac{\pi}{3}$

$\alpha = k\pi \pm \frac{\pi}{6}$

دقت کن !!

ب) $\cos(4\alpha - \frac{\pi}{6}) = \sin -2\alpha$ 2

$\cos(4\alpha - \frac{\pi}{6}) = \cos(\frac{\pi}{2} + 2\alpha)$ ① $4\alpha - \frac{\pi}{6} = 2k\pi + \frac{\pi}{2} + 2\alpha$

② $4\alpha - \frac{\pi}{6} = 2k\pi - \frac{\pi}{2} - 2\alpha \qquad 6\alpha = 2k\pi - \frac{2\pi}{6} \quad \alpha = \frac{2k\pi}{6} - \frac{2\pi}{36}$ ✓

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⑤ 1/5

$\frac{\sin 4\alpha + \sin 2\alpha}{\sin 2\alpha} = \underbrace{\cos^2\alpha + \sin^2\alpha}_{1} \qquad \sin 4\alpha = 0$

$\alpha = \frac{k\pi}{4}$ ——→ $\sin 2\alpha \neq 0 \qquad \alpha \neq \frac{k\pi}{2}$

شرط ... ...

$\alpha = \{\frac{k\pi}{4}\} - \{\frac{k\pi}{2}\}$

اگر $\sin 4\alpha = 0$ باشد در برخی از جواب ها $\sin 2\alpha$ هم صفر می‌شود، و عبارت تعریف نشده می‌شود

$\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x = 1$

$\frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \rightarrow \cos 2x = 0$

$2x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

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$$\frac{\sin\frac{\alpha}{2}}{1+\cos\frac{\alpha}{2}} = \frac{1}{\sin\frac{\alpha}{2}} + \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}$$

$\cos\frac{\alpha}{2}\neq -1$ ، $\sin\frac{\alpha}{2}\neq 0$

$$\left(\cos\frac{\alpha}{2}+1\right)^2 = \left(\sin\frac{\alpha}{2}\right)^2$$

$$1+\left(\cos\frac{\alpha}{2}\right)^2+2\cos\frac{\alpha}{2} = \sin^2\frac{\alpha}{2} \rightarrow 2\cos^2\frac{\alpha}{2}+2\cos\frac{\alpha}{2} = 0$$

$\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}$

$$2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}+1\right)=0$$

$\cos\frac{\alpha}{2}=0$ ✓ ← $\cos\frac{\alpha}{2}=-1$ → ✗ امکان‌پذیر نیست (خارج از زاویه)

$\cos\frac{\alpha}{2}=0 \quad \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ ✓

۲

$$\cos^2\alpha - \sin^2\alpha\cos 2\alpha = 1 \rightarrow \sin^2\alpha+\cos^2\alpha$$

$(\cos 2\alpha = 2\cos^2\alpha-1)$

استفاده از اتحاد مثلثاتی

$$\sin^2\alpha+\sin^2\alpha\left(2\cos^2\alpha-2\cos\alpha\right)=0$$

$$\sin^2\alpha\left(1+2\cos^2\alpha-2\cos\alpha\right)=0$$

$\sin\alpha=0$

$0, \pi, 2\pi$

$\cos 2\alpha=-1$

$\frac{\pi}{3}, \frac{5\pi}{3}$

در $[0,2\pi]$ جواب دارد

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$$\frac{1-\tan u}{1+\tan u} = \tan\left(\frac{\pi}{4}-u\right)$$

$\tan\frac{\pi}{4}$

$$\tan\left(\frac{\pi}{4}-u\right) = \tan 3u$$

$$3u = k\pi+\frac{\pi}{4}-u$$

$$4u = k\pi+\frac{\pi}{4}$$

$$u = \frac{k\pi}{4}+\frac{\pi}{16} \quad k\in\mathbb{Z}$$

دقت!!

$$u = \frac{k\pi}{4}+\frac{\pi}{16}$$

s.a.m

وقتی عبارت به توان ۲ می رسد ممکن است جواب زائد تولید کند!

(۹) 1,75

$\sqrt{1+\sin 2\alpha} = \sqrt{2}\cos 2\alpha \longrightarrow 1+\sin 2\alpha = 2\cos^2 2\alpha$

$\sin^2 2\alpha + \cos^2 2\alpha + \sin 2\alpha = 2\cos^2 2\alpha$

$2\sin^2 2\alpha + \sin 2\alpha - 1 = 0 \qquad 1-\sin^2 2\alpha$

جمع ضرایب اولی و آخری = وسطی $\qquad \sin 2\alpha = -1 \qquad \sin 2\alpha = 1/2$

$\sin 2\alpha = -1$ ✓ $\rightarrow \alpha = \frac{3\pi}{4}$ چون به توان رسوندیم جواب ها رو امتحان می کنیم

$\sin 2\alpha = 1/2$ ✓ $\rightarrow \alpha = \frac{\pi}{12}$, $\frac{5\pi}{12}$ ✗ ۲ جواب — جواب $\frac{5\pi}{12}$ $\cos 2x$ را منفی

می کند که غ ق ق است چون $\cos 2x$ جواب رادیکال است!

(۱۰) الف)

$\sin^4 x - \cos^4 x = 1$

$(\sin^2\alpha - \cos^2\alpha)(\sin^2\alpha + \cos^2 x) = \frac{1}{\sin^2 x + \cos^2\alpha}$

$2\cos^2 x = 0 \qquad \cos x = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$

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ب)

$\sin^3 x - \cos^3 x = -1$

$(\sin x - \cos x)(\sin^2 x + \cos^2 x + \sin x\cos x) = -1$

نقطه تقاطع
چک می کنیم

$0 \rightarrow 0 - 1 = -1$ ✓

$\frac{\pi}{2} \rightarrow 1 - 0 \neq -1$

$\pi \rightarrow 0 + 1 \neq -1$

$\frac{3\pi}{2} \rightarrow -1 + 0 = -1$

$2\pi \rightarrow 0 - 1 = -1$