السا خان محمدی

① الف) $\underbrace{\cos 2\alpha}_{1-2\sin^2\alpha} = 3\sin\alpha - 1 \Rightarrow 2\sin^2\alpha + 3\sin\alpha - 2 = 0$

$\sin^2\alpha + 3\sin\alpha - 4 = 0$

$(\sin\alpha + 4)(\sin\alpha - 1) = 0$

✗ $\sin = -2$ ✓ $\sin = 1/2$

$\sin = 1/2 \Rightarrow \boxed{2k\pi + \frac{\pi}{6}}$ و $\boxed{2k\pi + \frac{5\pi}{6}}$

ب) $\underbrace{\cos 2\alpha}_{2\cos^2\alpha - 1} + \cos\alpha - 2 = 0 \Rightarrow 2\cos^2\alpha + \cos\alpha - 3 = 0$ ✗ $\cos\alpha = -\frac{3}{2}$ ✓ $\cos\alpha = 1$

((جمع ضرایب = ۰))

$\cos\alpha = 1 \Rightarrow \boxed{2k\pi}$

② الف) $\sin^4\alpha - \cos^4\alpha = \sin 4\alpha$

$(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1})(\underbrace{\sin^2\alpha - \cos^2\alpha}_{-\cos 2\alpha})$

$\cos 2\alpha = \sin(-4\alpha)$

$\cos 2\alpha = \cos(\frac{\pi}{2} + 4\alpha)$

① $2\alpha = 2k\pi + \frac{\pi}{2} + 4\alpha$

$2\alpha = -2k\pi - \frac{\pi}{2}$ $\boxed{\alpha = -\frac{2k\pi - \frac{\pi}{2}}{2}}$

② $2\alpha = 2k\pi - \frac{\pi}{2} - 4\alpha$

$6\alpha = 2k\pi - \frac{\pi}{2}$ $\boxed{\alpha = \frac{2k\pi - \frac{\pi}{2}}{6}}$

ب) $\underbrace{2\cos^2\alpha - 1}_{\cos 2\alpha} = -\sin 2\alpha \Rightarrow \cos 2\alpha = \cos(\frac{\pi}{2} + 2\alpha)$

① $2\alpha = 2k\pi + \frac{\pi}{2} + 2\alpha$ ✗

② $2\alpha = 2k\pi - \frac{\pi}{2} - 2\alpha$ $4\alpha = 2k\pi - \frac{\pi}{2}$

$\boxed{\alpha = \frac{2k\pi - \frac{\pi}{2}}{4}}$

③ الف) $\underbrace{\cot}_{\frac{\cos\alpha}{\cancel{\sin\alpha}}}(2\cos\alpha + 1) = \frac{1}{\cancel{\sin\alpha}}$ $2\cos^2\alpha + \cos\alpha - 1 = 0 \rightarrow \cos = -1$

// جمع ضرایب زوجی و فردی = وسطی // $\Rightarrow \cos = 1/2$

① $\cos\alpha = -1 \Rightarrow \boxed{\alpha = 2k\pi + \pi}$ ② $\cos\alpha = 1/2 \Rightarrow \alpha = \boxed{2k\pi \pm \frac{\pi}{3}}$

★ شرط را هم داریم ، $\sin\alpha \neq 0$ پس این جواب غ ق ق

الیسا خان محمدی

ب) $\cancel{2\sin^2\alpha} - \sin\cos\alpha + \cancel{\cos^2\alpha} = \cancel{2\sin^2\alpha} + \cancel{2}\cos^2\alpha$

$\cos^2\alpha + \sin\cos\alpha = 0 \qquad \cos\alpha(\cos\alpha + \sin\alpha) = 0$

① $\cos\alpha = 0 \quad \boxed{\alpha = k\pi + \frac{\pi}{2}}$ ② $\cos\alpha = -\sin\alpha \rightarrow \boxed{\alpha = k\pi + \frac{3\pi}{4}}$

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④ الف) $\cos(\frac{\pi}{4} + \alpha) \cdot \cos(\alpha - \frac{\pi}{4}) = \frac{1}{4}$

$\cos(\frac{\pi}{4} - \alpha)$ + [illegible]

علامت منفی مشخص [illegible]

یکی منفی سینوس دیگه منفی [illegible]

$\boxed{\cos\alpha = \cos(-\alpha)}$ +

$\cos(\frac{\pi}{4} + \alpha) \times \sin(\frac{\pi}{4} + \alpha) = \frac{1}{4}$

$\frac{1}{2}\sin(\frac{\pi}{2} + 2\alpha) = \frac{1}{4}$

$\sin(\frac{\pi}{2} + 2\alpha) = \frac{1}{2} \qquad \cos 2\alpha = -\frac{1}{2} \qquad \boxed{\alpha = \frac{2k\pi + \frac{2\pi}{3}}{2}, \quad \alpha = \frac{2k\pi - \frac{2\pi}{3}}{2}}$

ب) $\cos(2\alpha - \frac{\pi}{9}) = \sin -2\alpha$

$\cos(2\alpha - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2\alpha)$ ① $2\alpha - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2\alpha$ ✗

② $2\alpha - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2\alpha \qquad 4\alpha = 2k\pi - \frac{7\pi}{18} \quad \boxed{\alpha = \frac{2k\pi - \frac{7\pi}{18}}{4}}$

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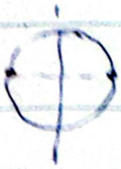
⑤ $\frac{\sin 4\alpha + \sin 2\alpha}{\sin 2\alpha} = \underbrace{\cos^2\alpha + \sin^2\alpha}_{1} \qquad \sin 4\alpha = 0$



$\alpha = \frac{k\pi}{4} \xrightarrow[\text{[illegible]}]{} \sin 2\alpha \neq 0 \qquad \alpha \neq \frac{k\pi}{2}$

$\alpha = \{\frac{k\pi}{4}\} - \{\frac{k\pi}{2}\}$

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$\cos\frac{\alpha}{2} \neq -1$

$$\frac{\sin\frac{\alpha}{2}}{1+\cos\frac{\alpha}{2}} = \frac{1}{\sin\frac{\alpha}{2}} + \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} \quad \sin\frac{\alpha}{2} \neq 0$$

$$(\cos\frac{\alpha}{2}+1)^2 = (\sin\frac{\alpha}{2})^2$$

$$1 + (\cos\frac{\alpha}{2})^2 + 2\cos\frac{\alpha}{2} = \cancel{\sin^2\frac{\alpha}{2}} \rightarrow 2\cos^2\frac{\alpha}{2} + 2\cos\frac{\alpha}{2} = 0$$

$$\cancel{\sin^2\frac{\alpha}{2}} + \cos^2\frac{\alpha}{2}$$

$$2\cos\frac{\alpha}{2}(\cos\frac{\alpha}{2}+1) = 0$$

✓ $0 = \cos\frac{\alpha}{2}$ $\quad$ $\cos\frac{\alpha}{2} = -1$ ✗ امکان پذیر نیست (خارج از دامنه)

$$\cos\frac{\alpha}{2} = 0 \quad \boxed{\left\{\frac{\pi}{[illegible]}, \frac{3\pi}{[illegible]}\right\}}$$

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$$\cancel{\cos^2\alpha} - \sin^2\alpha \underbrace{\cos 3\alpha}_{(4\cos^3\alpha - 3\cos\alpha)} = 1 \rightarrow \sin^2 + \cancel{\cos^2\alpha}$$

اشتباه کردم بنابراین پاک کردم

$$\sin^2\alpha + \sin^2\alpha(4\cos^3\alpha - 3\cos\alpha) = 0$$

$$\sin^2\alpha(1 + 4\cos^3\alpha - 3\cos\alpha) = 0$$

$$\boxed{\cos 3\alpha = -1} \quad \frac{\pi}{3}, \frac{5\pi}{3}$$

$\rightarrow \sin\alpha = 0$

$$\boxed{0, 2\pi, \pi}$$

در کل ۵ جواب دارد

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$$\frac{1 - \tan x}{1 + \tan x} = \tan 3x \quad (1 = \tan\frac{\pi}{4})$$

$$\tan(\frac{\pi}{4} - x) = \tan 3x$$

$$3x = k\pi + \frac{\pi}{4} - x \qquad \boxed{x = \frac{k\pi}{4} + \frac{\pi}{16} \quad k \in \mathbb{Z}}$$

s.a.m

⑨

$\sqrt{1+\sin 2\alpha} = \sqrt{2}\cos 2\alpha \rightarrow 1+\sin 2\alpha = 2\cos^2 2\alpha$

$\sin 2\alpha + \cancel{\cos^2 2\alpha} + \sin^2 2\alpha = \cancel{2}\cos^2 2\alpha$ ~~$(1-2\sin^2\alpha)$~~

$2\sin^2 2\alpha + \sin 2\alpha - 1 = 0$ $\quad 1-\sin^2 2\alpha$

جمع ضرایب اولی و آخری = وسطی $\quad \sin 2\alpha = -1 \quad \sin 2\alpha = 1/2$

$\sin 2\alpha = -1$ ✓ $\rightarrow \boxed{\alpha = \frac{3\pi}{4}}$ چون به توان رسوندیم جواب هامون و امتحان می کنیم

$\sin 2\alpha = 1/2$ ✓ $\rightarrow \boxed{\alpha = \frac{\pi}{12}, \frac{5\pi}{12}}$ ۲ جواب

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⑩

الف) $\sin^4 x - \cos^4 x = 1$

$(\sin^2\alpha - \cos^2\alpha)\underbrace{(\sin^2\alpha + \cos^2 x)}_{1} = \frac{1}{\cancel{\sin^2 x + \cos^2\alpha}}$

$2\cos^2 x = 0 \quad \cos x = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$

$(\sin x - \cos x)(\underbrace{\sin^2 x + \cos^2 x}_{1} + \sin x\cos x) = -1$

۴ تقاطع

چک می کنیم

$0 \rightarrow 0 - 1 = -1$ ✓ $\qquad \frac{3\pi}{2} \quad -1 + 0 = -1$

$\frac{\pi}{2} \rightarrow 1 - 0 \neq -1 \qquad 2\pi \rightarrow 0 - 1 = -1$

$\pi \rightarrow -0 + 1 \neq -1$