دوازدهم ریاضی شیب ۱۲

۱۸,۷۵

۱- الف) $\cos 2x + \cos x - 2 = 0$

$2\cos^2 x - 1 + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0$

$\cos^2 x + \cos x - 2 = 0$

$(\cos x + \frac{3}{2})(\cos x - 1) = 0$

$\cos x = -\frac{3}{2}$ غیر قابل قبول $\quad \cos x = 1$ قابل قبول

$\cos x = 1 \Rightarrow x = 2k\pi$

۲

$\cos 2x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1$

$4\sin^2 x + 3\sin x - 2 = 0$

$\sin^2 x + 3\sin x - 4 = 0$

$(\sin x + 4)(\sin x - 1) = 0$

$-2 = -\frac{4}{2}$ غیر قابل قبول $\quad \frac{1}{2}$

$\sin x = \frac{1}{2} \to x \in 2k\pi + \frac{\pi}{6}$

$\to x \in 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6}$

۲- الف) $\sin^4 x - \cos^4 x = \sin 4x$

$\underbrace{(\sin^2 x + \cos^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = \sin 4x$

$\to \sin 4x = -\cos 2x$

$2\sin 2x \cancel{\cos 2x} = -\cancel{\cos 2x} \to \cos 2x = 0$

$2\sin 2x = -1 \to \sin 2x = -\frac{1}{2} \quad \frac{7\pi}{6}$

۲

$2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12}$

$2x = 2k\pi + \pi - \frac{7\pi}{6}$

$x = k\pi - \frac{\pi}{12}$

$\cos 2x = 0$

$2x \in 2k\pi \pm \frac{\pi}{2} \to x \in k\pi \pm \frac{\pi}{4}$

ب) $2\cos^2 x + \underbrace{2\sin x\cos x}_{\sin 2x} = 1$

$\cancel{1} + \cos 2x + \sin 2x = \cancel{1}$

$\cos 2x = -\sin 2x$

$2x \in 2k\pi + \frac{3\pi}{4} \to x = k\pi + \frac{3\pi}{8}$

$2x \in 2k\pi + \frac{7\pi}{4} \to x = k\pi + \frac{7\pi}{8}$

۳-

$\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x}$

$\frac{\cos x}{\sin x}(2\cos x + 1) - \frac{1}{\sin} = 0 \to \frac{1}{\sin}(2\cos^2 x + \cos x - 1) = 0$

$\sin \neq 0$

$\cos^2 x + \cos x - 2 = 0$

$(\cos x + 2)(\cos x - 1) = 0$

۲

غ ق $\frac{-2}{2} = -1 \quad \frac{1}{2}$ ق

چون اگر $\cos x = -1$، $\sin x = 0$ و با توجه به شرط خارج قابل قبول نیست

$\cos x = \frac{1}{2} \quad \frac{\pi}{3} \to x = 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$

$\underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{\sin^2 x - \sin x\cos x}_{1} = 2$

$\sin^2 x - \sin x\cos x = \cos^2 x + \sin^2 x$

$\cos^2 x + \sin x\cos x = 0$

$\cos x(\cos x + \sin x) = 0$

$\cos x = 0 \quad \sin x = -\cos x$

$x \in 2k\pi + \frac{\pi}{2}$

$x \in 2k\pi + \frac{3\pi}{2}$

$x \in 2k\pi + \frac{7\pi}{4}$

۴ - 1

قسمت ب ؟

$$\cos(\frac{\pi}{4}+x)\,\cos(x-\frac{\pi}{4})=\frac{1}{4}$$

$$\cos(x-\frac{\pi}{4})=\cos(\frac{\pi}{4}-x)$$

$$\frac{\pi}{4}-x+\frac{\pi}{4}+x=\frac{\pi}{2}\rightarrow$$ دو زاویه متمم

cos و sin a

برابر است.

$$\sin(x+\frac{\pi}{4})\,\cos(x+\frac{\pi}{4})=\frac{1}{4}$$

$$\frac{1}{2}\sin(2x+\frac{\pi}{2})=\frac{1}{4}$$

$$\sin(2x+\frac{\pi}{2})=\cancel{\frac{1}{2}}\ \frac{\pi}{6}$$

$$2x+\frac{\pi}{2}=2k\pi+\frac{\pi}{3}\rightarrow x\in k\pi+\frac{\pi}{6}$$

$$2x+\frac{\pi}{2}=2k\pi-\frac{\pi}{3}\rightarrow x\in k\pi-\frac{\pi}{6}$$

$$x\in k\pi\pm\frac{\pi}{6}$$

۵ -

$$\frac{\sin 4x+\sin 2x}{\sin 2x}=\overbrace{\sin^2 x+\cos^2 x}^{1}$$

$$\sin 4x+\sin\cancel{2x}=\sin\cancel{2x}$$

$$\rightarrow\frac{2\sin 2x\cos 2x+\sin 2x}{\sin 2x}=1$$

$$\frac{\sin 2x(2\cos 2x+1)}{\cancel{\sin 2x}}=2\cos^2 x+1$$

$$2\cos 2x+1=1$$

$$\cos 2x=0\rightarrow 2x\in k\pi+\frac{\pi}{2}$$

$$x\in k\pi+\frac{\pi}{4}$$

2

$x \in k\pi \pm \frac{\pi}{6}$

-۵

$$\frac{\sin 4x + \sin 2x}{\sin 2x} = \cancel{\sin^2 x + \cos^2 x}^{\,1}$$

$\sin 4x + \cancel{\sin 2x} = \cancel{\sin 2x}$

$$\to \frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = 1$$

$$\frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} = 2\cos 2x + 1$$

$2\cos 2x + 1 = 1$

$\cos 2x = 0 \to 2x \in 2k\pi + \frac{\pi}{2}$

$x \in k\pi + \frac{\pi}{4}$

-۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2}$$

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1-\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \tan\frac{x}{4} = \frac{1}{\tan\frac{x}{4}}$$

$\tan^2\frac{x}{4} = 1 \to \tan\frac{x}{4} = \pm 1 \to \frac{x}{4} \in k\frac{\pi}{4} \pm \frac{\pi}{4}$

$x \in 4k\pi \pm \pi$

$-\pi \leq x \leq \frac{5\pi}{2}$ $\Big\} \to x = -\pi, \pi$

(تفاضل) $= \pi - (-\pi) = 2\pi$

-۷

$\cos^4 x - \sin^2 x \cos^2 x = 1$

$\cos^2 x - 1 = \sin^2 x$

$\cancel{1} - \sin^2 x - \sin^2 x \cos^2 x = \cancel{1}$

$-\sin^2 x(1+\cos^2 x) = 0$

$\sin x = 0 \to x \in k\pi$, $\cos^2 x = -1$

$x \in 2k\pi + \pi$

$x \in \frac{k\pi}{2} + \frac{\pi}{4}$

$[0, 2\pi] \to 0, \pi, 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}$ ۵ جواب

۸-

$$\frac{1-\tan x}{1+\tan x}=\tan 3x \rightarrow \frac{\tan\frac{\pi}{4}-\tan x}{1+(\tan\frac{\pi}{4}\times\tan x)}=\tan 3x$$

$$\tan(\frac{\pi}{4}-x)=\tan 3x \rightarrow$$

$$3x=k\pi+\frac{\pi}{4}-x$$

$$4x=k\pi+\frac{\pi}{4}\rightarrow x=\frac{k\pi}{4}+\frac{\pi}{16}$$

۲

۹-

$$\sqrt{1+\sin 2x}=\sqrt{2}\cos 2x \qquad [0,\pi]$$

$$\cos 2x>0$$

$$1+\sin 2x=2\cos^2 2x \longrightarrow 1+\sin 2x=2-2\sin^2 2x$$

$$2\sin^2 2x+\sin 2x-1=0$$

$$\sin^2 2x+\sin 2x-2=0$$

$$(\sin 2x+2)(\sin 2x-1)=0$$

$$\sin 2x=\frac{-2}{2} \qquad \sin 2x=\frac{1}{2}$$

۱,۲۵

$$\sin 2x=-1\rightarrow 2x\in 2k\pi+\frac{3\pi}{2}\rightarrow x\in k\pi+\frac{3\pi}{4}$$

$$\sin 2x=\frac{1}{2}\rightarrow 2x\in 2k\pi+\frac{\pi}{6}\rightarrow x\in k\pi+\frac{\pi}{12}$$

$$2x\in 2k\pi+\frac{5\pi}{6}\rightarrow x\in k\pi+\frac{5\pi}{12}\longrightarrow[0,\pi]\rightarrow\frac{3\pi}{4},\frac{\pi}{12},\frac{5\pi}{12}$$

۳ جواب

به ازای $x=\frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، یک جواب را اشکال است؟ نمی‌شود!

۱۰- $[0,2\pi]$

الف) $\sin^4 x-\cos^4 x=1$

$$\cancel{(\sin^2 x+\cos^2 x)}(\sin^2 x-\cos^2 x)=1$$

$$-\cos 2x=1\rightarrow\cos 2x=-1\rightarrow 2x=2k\pi+\pi$$

$$x=k\pi+\frac{\pi}{2}\rightarrow\frac{\pi}{2},\frac{3\pi}{2}$$

۲

از مقادیر آزمایشی در مثلثات استفاده می‌کنیم

ب) $\sin^3 x-\cos^3 x=-1$

$x=0\longrightarrow 0-1=-1$ ✓

$x=\frac{\pi}{2}\rightarrow 1-0=1$ ✗

$x=\pi\rightarrow 0-(-1)=1$ ✗

$x=\frac{3\pi}{2}\Rightarrow -1-0=-1$ ✓

$x=2\pi\rightarrow 0-1=-1$ ✓

در بازه $\rightarrow[0,2\pi]\longrightarrow x\in 0,\frac{3\pi}{2},2\pi$

سوال۴ قسمت ب

$$\cos\left(2x - \frac{\pi}{9}\right) = -\sin 2x$$

$$\cos\left(2x - \frac{\pi}{9}\right) = \cos\left(\frac{\pi}{2} + 2x\right)$$

$$2x - \frac{\pi}{9} = 2k\pi \pm \left(\frac{\pi}{2} + 2x\right)$$

حالت $+$: $2k\pi + \frac{\pi}{9} + \frac{\pi}{2} = 0$ ✗

حالت $-$: $2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x$

$$4x = 2k\pi - \frac{7\pi}{18} \rightarrow \boxed{x = \frac{k\pi}{2} - \frac{7\pi}{72}}$$