دوازدهم ریاضی شیب ۱۴ — پاسخ تمرین (تکالیف متوسط)

۱-

الف) $\cos 2x + \cos x - 2 = 0$

$2\cos^2 x - 1 + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0$

$\cos^2 x + \cos x - 6 = 0$

$(\cos x + 3)(\cos x - 2) = 0$

$\cos x = \frac{-3}{2}$ غیر قابل قبول — $\cos x = \frac{2}{2} = 1$ قابل قبول

$\cos x = 1 \Rightarrow x = 2k\pi$

ب) $\cos 2x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1$

$4\sin^2 x + 3\sin x - 2 = 0$

$\sin^2 x + 3\sin x - 4 = 0$

$(\sin x + 4)(\sin x - 1) = 0$

$\frac{-4}{2} = -2$ غیر قابل قبول — $\frac{1}{2}$

$\sin x = \frac{1}{2} \to x \in 2k\pi + \frac{\pi}{6}$

$\to x \in 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6}$

۲-

الف) $\sin^4 x - \cos^4 x = \sin 4x$

$\underbrace{(\sin^2 x + \cos^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = \sin 4x$

$\to \sin 4x = -\cos 2x$

$2\sin 2x \cos 2x = -\cos 2x \to \cos 2x = 0$

$2\sin 2x = -1 \to \sin 2x = -\frac{1}{2}$ ، $\frac{7\pi}{6}$

$2x = 2k\pi + \frac{7\pi}{6} \to 2x = k\pi + \frac{7\pi}{12}$

$2x = 2k\pi + \pi - \frac{7\pi}{6}$

$x = k\pi - \frac{\pi}{12}$

$\cos 2x = 0$

$2x \in 2k\pi \pm \frac{\pi}{2} \to x \in k\pi \pm \frac{\pi}{4}$

ب) $2\cos^2 x + \underbrace{2\sin x\cos x}_{\sin 2x} = 1$

$\cancel{1} + \cos 2x + \sin 2x = \cancel{1}$

$\cos 2x = -\sin 2x$

$2x \in 2k\pi + \frac{3\pi}{4} \to x = k\pi + \frac{3\pi}{8}$

$2x \in 2k\pi + \frac{7\pi}{4} \to x = k\pi + \frac{7\pi}{8}$

۳-

$\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x}$

$\frac{\cos x}{\sin x}(2\cos x + 1) - \frac{1}{\sin} = 0 \to \frac{1}{\sin}(2\cos^2 x + \cos x - 1) = 0$

$\sin \neq 0$

$\cos^2 x + \cos x - 2 = 0$

$(\cos x + 2)(\cos x - 1) = 0$

$\frac{-2}{2} = -1$ ق ق — $\frac{1}{2}$ ق

اگر $\cos x = -1$ ، $\sin x = 0$ و با توجه به شرط غیر قابل قبول است.

$\cos x = \frac{1}{2}$ ، $\frac{\pi}{3} \to x = 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$

$\underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{\sin^2 x - \sin x\cos x}_{1} = 2$

$\sin^2 x - \sin x\cos x = \cos^2 x + \sin^2 x$

$\cos^2 x + \sin x\cos x = 0$

$\cos x(\cos x + \sin x) = 0$

$\cos x = 0$ ، $\sin x = -\cos x$

$x \in 2k\pi + \frac{\pi}{2}$

$x \in 2k\pi + \frac{3\pi}{4}$

$x \in 2k\pi + \frac{7\pi}{4}$

۴-

$\cos(\frac{\pi}{4}+x)\ \cos(x-\frac{\pi}{4}) = \frac{1}{4}$

$\cos(x-\frac{\pi}{4}) = \cos(\frac{\pi}{4}-x)$

$\frac{\pi}{4} - x + \frac{\pi}{4} + x = \frac{\pi}{2} \to$ دو زاویه‌ی متمم

$\cos(90-\alpha)$ و $\sin\alpha$

برابر است.

$\sin(x+\frac{\pi}{4})\ \cos(x+\frac{\pi}{4}) = \frac{1}{4}$

$\frac{1}{2}\sin(2x+\frac{\pi}{2}) = \frac{1}{4}$

$\sin(2x+\frac{\pi}{2}) = \cancel{\frac{1}{2}}\ \frac{\pi}{6}$

$2x+\frac{\pi}{2} = 2k\pi + \frac{\pi}{3} \to x \in k\pi + \frac{\pi}{6}$

$2x+\frac{\pi}{2} = 2k\pi - \frac{\pi}{3} \to x \in k\pi - \frac{\pi}{6}$

$x \in k\pi \pm \frac{\pi}{6}$

۵-

$\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x \overset{1}{}$

$\sin 4x + \sin \cancel{2x} = \sin \cancel{2x}$

$\to \frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = 1$

$\frac{\sin 2x(2\cos 2x + 1)}{\cancel{\sin 2x}} = 2\cos 2x + 1$

$2\cos 2x + 1 = 1$

$\cos 2x = 0 \to 2x \in k\pi + \frac{\pi}{2}$

$x \in k\pi + \frac{\pi}{4}$

$x \in k\pi \pm \frac{\pi}{6}$

۵ -

$\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x = 1$

$\sin 4x + \cancel{\sin 2x} = \cancel{\sin 2x}$

$\rightarrow \frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = 1$

$\frac{\sin 2x(2\cos 2x + 1)}{\cancel{\sin 2x}} = 2\cos 2x + 1$

$2\cos 2x + 1 = 1$

$\cos 2x = 0 \rightarrow 2x \in 2k\pi + \frac{\pi}{2}$

$x \in k\pi + \frac{\pi}{4}$

۶ -

$\frac{\sin \frac{x}{2}}{1+\cos \frac{x}{2}} = \frac{1}{\sin \frac{x}{2}} + \cot \frac{x}{2}$

$\frac{\sin \frac{x}{2}}{1+\cos \frac{x}{2}} = \frac{1-\cos \frac{x}{2}}{\sin \frac{x}{2}} \Rightarrow \tan \frac{x}{4} = \frac{1}{\tan \frac{x}{4}}$

$\tan^2 \frac{x}{4} = 1 \rightarrow \tan \frac{x}{4} = \pm 1 \rightarrow \frac{x}{4} \in \frac{k\pi}{4} \pm \frac{\pi}{4}$

$x \in 2k\pi \pm \pi$

$-\pi \leq x \leq \frac{5\pi}{2}$ $\Big\} \rightarrow x = -\pi, \pi$

$(\text{مجموع} = \pi - (-\pi)$

$= 2\pi$

۷ -

$\cos^4 x - \sin^2 x \cos^2 x = 1$ $\qquad \cancel{1} - \sin^2 x - \sin^2 x \cos^2 x = \cancel{1}$

$\cos^2 x - 1 = \sin^2 x$ $\qquad -\sin^2 x(1 + \cos^2 x) = 0$

$\sin x = 0$ $\qquad \cos^2 x = -1$

$x \in k\pi$ $\qquad x \in 2k\pi + \pi$

$x \in \frac{k\pi}{2} + \frac{\pi}{2}$

$[0, 2\pi] \rightarrow 0, \pi, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}$ ۵ جواب دارد

۸-

$$\frac{1-\tan x}{1+\tan x} = \tan^3 x \rightarrow \frac{\tan\frac{\pi}{4} - \tan x}{1 + (\tan\frac{\pi}{4} \times \tan x)} = \tan^3 x$$

$$\tan(\frac{\pi}{4} - x) = \tan^3 x \rightarrow$$

$$3x = k\pi + \frac{\pi}{4} - x$$

$$4x = k\pi + \frac{\pi}{4} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$$

۹-

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \qquad [0, \pi]$$

$$\cos 2x > 0$$

$$1 + \sin 2x = 2\cos^2 2x \longrightarrow 1 + \sin 2x = 2 - 2\sin^2 2x$$

$$2\sin^2 2x + \sin 2x - 1 = 0$$

$$\sin^2 2x + \sin 2x - 2 = 0$$

$$(\sin 2x + 2)(\sin 2x - 1) = 0$$

$$\sin 2x = \frac{-2}{2} \qquad \sin 2x = \frac{1}{2}$$

$$\sin 2x = -1 \rightarrow 2x \in 2k\pi + \frac{3\pi}{2} \rightarrow x \in k\pi + \frac{3\pi}{4}$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x \in 2k\pi + \frac{\pi}{6} \rightarrow x \in k\pi + \frac{\pi}{12}$$

$$\hookrightarrow 2x \in 2k\pi + \frac{5\pi}{6} \rightarrow x \in k\pi + \frac{5\pi}{12}$$

$$\longrightarrow [0, \pi] \rightarrow \frac{3\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$$

۳ جواب

۱۰- $[0, 2\pi]$

الف) $\sin^4 x - \cos^4 x = 1$

$$(\sin^2 x + \cancel{\cos^2 x})(\sin^2 x - \cos^2 x) = 1$$

$$-\cos 2x = 1 \rightarrow \cos 2x = -1 \rightarrow 2x = 2k\pi + \pi$$

$$x = k\pi + \frac{\pi}{2} \rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$$

از مقادیر آزمایشی در مثلث مثلثات استفاده می‌کنیم

ب) $\sin^3 x - \cos^3 x = -1$

$x = 0 \longrightarrow 0 - 1 = -1$ ✓

$x = \frac{\pi}{2} \rightarrow 1 - 0 = 1$ ✗

$x = \pi \rightarrow 0 - (-1) = 1$ ✗

$x = \frac{3\pi}{2} \Rightarrow -1 - 0 = -1$ ✓

$x = 2\pi \rightarrow 0 - 1 = -1$ ✓

در بازه

$$\rightarrow [0, 2\pi] \longrightarrow x \in 0, \frac{3\pi}{4}, 2\pi$$

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