Date: .... آفرین ۱۹٫۲۵ Subject: ....

۱- الف) $\cos 2x = 3\sin x - 1$ $\quad 2\sin^2 x = 1 - \cos 2x$ روژین قسوریان (۱۶۲)

$\rightarrow 1 - 2\sin^2 x = 3\sin x - 1$ $\quad 2\sin^2 x + 3\sin x - 2 = 0$

$\sin^2 x + \frac{3}{2}\sin x - 1 = 0 \rightarrow \sin x = -2$ غ ق ق $\quad \sin x = \frac{1}{2}$ ✓ ۲

جواب کلی: $x = 2k\pi + \frac{\pi}{6}$, $x = 2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0$ $\quad 2\cos^2 x = 1 + \cos 2x$

$\rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\cos^2 x + \cos x - 6 = 0$ $\quad (\cos x + 3)(\cos x - 2) = 0 \rightarrow \cos x = +1$ $\quad \cos x = -\frac{3}{2}$ غ ق ق

$x = 2k\pi$

۲- الف) $\sin^4 x - \cos^4 x = \sin 4x$ $\quad (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = \sin 4x$

$-\cos 2x = -\sin 4x \rightarrow \cos 2x = \sin 4x \rightarrow \cos 2x = \cos(\frac{\pi}{2} + 4x)$ ۲

$2x = 2k\pi + \frac{\pi}{2} + 4x \rightarrow -2x = \frac{\pi}{2}(4k+1) \rightarrow x = -\frac{\pi}{4}(4k+1)$

$2x = 2k\pi - \frac{\pi}{2} - 4x \rightarrow 6x = \frac{\pi}{2}(4k-1) \rightarrow x = \frac{\pi}{12}(4k-1)$

ب) $2\cos^2 x + 2\sin x\cos x - 1 = 0 \rightarrow 1 + \cos 2x + \sin 2x = 1$

$\rightarrow \cos 2x = -\sin 2x \rightarrow \cos 2x = \sin(-2x) \rightarrow \cos 2x = \cos(\frac{\pi}{2} + 2x)$

$\cancel{2x} = 2k\pi + \frac{\pi}{2} + \cancel{2x} \rightarrow \cancel{k = -\frac{\pi}{4}}$

$2x = 2k\pi - \frac{\pi}{2} - 2x \rightarrow 4x = \frac{\pi}{2}(4k-1) \rightarrow x = \frac{\pi}{8}(4k-1)$

۳- الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x}$ $\quad \sin x \neq 0$

$\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \rightarrow 2\cos^2 x + \cos x = 1 \rightarrow 2\cos^2 x + \cos x - 1 = 0$ ۲

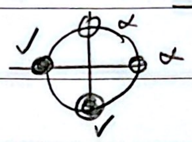
$\frac{-1 \pm \sqrt{1+8}}{4} = \rightarrow \cos x = -1, \frac{1}{2}$ غ ق ق $\rightarrow x = 2k\pi + \frac{\pi}{3}, 2k\pi - \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$ $2\sin^2 x = 1 - \cos 2x$ (*)

$(\sin^2 x + \cancel{\cos^2 x}) + \sin^2 x - \sin x\cos x = 2$

$\sin^2 x - \sin x\cos x - 1 = 0 \xrightarrow{\times 2} 2\sin^2 x - \sin 2x - 2 = 0$ (*)

$1 - \cos 2x - \sin 2x - 2 = 0 \longrightarrow -1 = \sin 2x + \cos 2x$



$\longrightarrow 2x = 2k\pi + \pi \longrightarrow x = \frac{\pi}{2}(2k+1)$

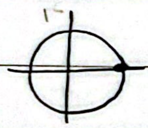
$\longrightarrow 2x = 2k\pi + \frac{3\pi}{2} \longrightarrow x = \frac{\pi}{4}(4k+3)$

۴- $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4}$

$(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x)(\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\cos x) = \frac{1}{4}$

$(\cos\frac{\pi}{4}\cos x)^2 - (\sin\frac{\pi}{4}\sin x)^2 = \frac{1}{4} \longrightarrow (\frac{\sqrt{2}}{2}\cos x)^2 - (\frac{\sqrt{2}}{2}\sin x)^2 = \frac{1}{4}$

$\frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x = \frac{1}{4} \longrightarrow \cos^2 x - \sin^2 x = 1 \longrightarrow \cos 2x = 1$



$2x = 2k\pi \longrightarrow x = k\pi$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x$

$\cos(2x - \frac{\pi}{9}) = \sin(-2x) \longrightarrow \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x$ غ ق ق

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \longrightarrow 4x = \frac{\pi}{18}(36k - 9 + 2) =$

$\longrightarrow x = \frac{\pi}{72}(36k - 7)$

۵- $\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$

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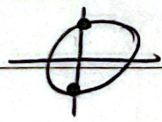
$\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 - \sin^2 x = \cos^2 x$

$2\cos 2x + \cancel{\cos^2 x} = \cancel{\cos^2 x} \longrightarrow \cos 2x = 0$



$2x = k\pi + \frac{\pi}{2}$

$2x = \frac{\pi}{2}(2k+1) \longrightarrow x = \frac{\pi}{4}(2k+1)$

6 _ $\dfrac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \dfrac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \Rightarrow \dfrac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \dfrac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

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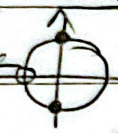
$\sin\frac{x}{2} \neq 0$ $\quad \sin^2\frac{x}{2} = 1 + 2\cos\frac{x}{2} + \cos^2\frac{x}{2} \longrightarrow \cos^2\frac{x}{2} + 2\cos\frac{x}{2} + \left(1-\sin^2\frac{x}{2}\right) = 0$

$\cos\frac{x}{2} \neq -1$ $\longrightarrow 2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \qquad \cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right) = 0$



$\longrightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \longrightarrow x = 2k\pi + \pi \Rightarrow x = \pi(2k+1)$

$-\pi \le \pi(2k+1) \le \frac{3\pi}{2} \longrightarrow -1 \le 2k+1 \le \frac{3}{2} \longrightarrow -2 \le 2k \le \frac{1}{2}$

$-1 \le k \le \frac{1}{4} \longrightarrow k = \boxed{0, -1} \longrightarrow x = \pi, -\pi \longrightarrow |x_1 - x_2| = 2\pi$

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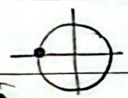
7 _ $\cos^2 x - \sin^2 x \cos 3x = 1$

$(\cos^2 x - 1) - \sin^2 x \cos 3x = 0 \longrightarrow \sin^2 x + \sin^2 x \cos 3x = 0$

$\sin^2 x(1+\cos 3x) = 0$



$3x = 2k\pi + \pi \longrightarrow x = \frac{\pi}{3}(2k+1)$

① $x = k\pi \longrightarrow$

$\longrightarrow$ هم جواب

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8 _ $\dfrac{1-\tan x}{1+\tan x} = \tan 3x \longrightarrow \dfrac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\tan x} = \tan 3x$

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$\longrightarrow \tan\left(\frac{\pi}{4} - x\right) = \tan 3x \longrightarrow 3x = k\pi + \frac{\pi}{4} - x \longrightarrow 4x = \frac{\pi}{4}(4k+1)$

$\longrightarrow x = \frac{\pi}{16}(4k+1)$

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9 _ $\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \qquad [0\ \pi]$ $\qquad 2\cos^2 2x = 1+\cos 4x$

$\sqrt{(\cos x + \sin x)^2} = \sqrt{2}\cos 2x \longrightarrow \cos^2 x + \sin^2 x + 2\sin x\cos x = 2\cos^2 2x$

$1 - 2\cos^2 2x + \sin 2x = 0 \longrightarrow \cos 4x = \sin 2x$

$\cos 4x = \cos\left(\frac{\pi}{2} - 2x\right) \qquad 4x = 2k\pi + \frac{\pi}{2} - 2x \longrightarrow 6x = \frac{\pi}{12}(4k+1)$

$4x = 2k\pi - \frac{\pi}{2} + 2x \longrightarrow 2x = \frac{\pi}{4}(4k-1)$

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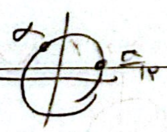
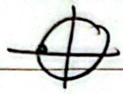


$0 \le 2x \le \frac{\pi}{2}$

$x \le \frac{\pi}{4}$

Date: ۱۰ Subject:

الف) $(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)=1 \to -\cos 2x=1$

$\cos 2x=-1$  $2x=2k\pi+\pi \to x=\frac{\pi}{2}(2k+1)$

$\pi(2k+1)$

$\to x=\frac{\pi}{2}, \frac{3\pi}{2}$ جواب ۲

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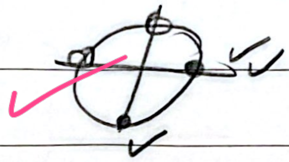
ب) $(\sin x-\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)=-1$

~~$\sqrt{2}\sin(x-\frac{\pi}{4})(1-\frac{1}{2}\sin 2x)=-1$~~  ۳ جواب

سوال ۴ تست الف ۱

$\cos(-x)=\cos x$

$\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} = \cos(\frac{\pi}{4}+x)\cos(\frac{\pi}{4}-x)=\frac{1}{4}$

if $\alpha+\beta=\frac{\pi}{2} \to \cos\alpha=\sin\beta \to \cos(\frac{\pi}{4}+x)\sin(\frac{\pi}{4}+x)=\frac{1}{4}$

$\frac{1}{2}\sin(\frac{\pi}{2}+2x)=\frac{1}{4} \to \cos 2x=\frac{1}{2} \to 2x=2k\pi \pm \frac{\pi}{3}$

$\boxed{x=k\pi \pm \frac{\pi}{6}}$