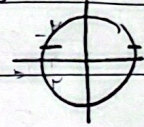


١- الف) $\cos 2x = 3 \sin x - 1$ $2 \sin^2 x = 1 - \cos 2x$ (١٤) $\sin^2 x = \frac{1 - \cos 2x}{2}$

$\rightarrow 1 - 2 \sin^2 x = 3 \sin x - 1$ $2 \sin^2 x + 3 \sin x - 2 = 0$

$\sin^2 x + \frac{3}{2} \sin x - 1 = 0 \rightarrow \sin x = -\frac{3}{2} \text{ أو } \frac{1}{2}$ $\sin x = \frac{1}{2}$ ✓



جواب: $x = 2K\pi + \frac{\pi}{6}$, $x = 2K\pi + \frac{5\pi}{6}$

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ب) $\cos 2x + \cos x - 2 = 0$ $2 \cos^2 x = 1 + \cos 2x$

$\rightarrow 2 \cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2 \cos^2 x + \cos x - 3 = 0$

$\cos^2 x + \cos x - \frac{3}{2} = 0$ $(\cos x + \frac{3}{2})(\cos x - 1) = 0 \rightarrow \cos x = -\frac{3}{2} \text{ أو } 1$

$x = 2K\pi$

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٢- الف) $\sin^2 x - \cos^2 x = \sin 2x$ $(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = \sin 2x$

$\cos 2x = -\sin 2x \rightarrow \cos 2x = \sin 2x \rightarrow \cos 2x = \cos(\frac{\pi}{2} + 2x)$

$2x = 2K\pi + \frac{\pi}{2} + 2x \rightarrow -2x = \frac{\pi}{2} (2K+1) \rightarrow x = -\frac{\pi}{4} (2K+1)$

$2x = 2K\pi - \frac{\pi}{2} + 2x \rightarrow 0 = \frac{\pi}{2} (2K-1) \rightarrow x = \frac{\pi}{4} (2K-1)$

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ب) $2 \cos^2 x + 2 \sin x \cos x - 1 = 0 \rightarrow 1 + \cos 2x + \sin 2x = 1$

$\rightarrow \cos 2x = -\sin 2x \rightarrow \cos 2x = \sin(-2x) \rightarrow \cos 2x = \cos(\frac{\pi}{2} + 2x)$

$2x = 2K\pi + \frac{\pi}{2} + 2x \rightarrow K = -\frac{\pi}{2}$

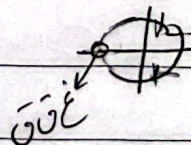
$2x = 2K\pi - \frac{\pi}{2} - 2x \rightarrow 4x = \frac{\pi}{2} (2K-1) \rightarrow x = \frac{\pi}{8} (2K-1)$

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٣- الف) $\cot x (2 \cos x + 1) = \frac{1}{\sin x}$ $\sin x \neq 0$

$\frac{\cos x}{\sin x} (2 \cos x + 1) = \frac{1}{\sin x} \rightarrow 2 \cos^2 x + \cos x = 1 \rightarrow 2 \cos^2 x + \cos x - 1 = 0$

$\frac{-1 \pm \sqrt{1+8}}{4} = \cos x = -1, \frac{1}{2}$ $x = 2K\pi + \frac{\pi}{2}, 2K\pi - \frac{\pi}{2}$

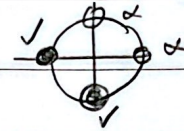


$$b) r \sin^2 x - \sin x \cos x + \cos^2 x = r \quad r \sin^2 x = 1 - \cos^2 x \quad (*)$$

$$(r \sin^2 x + \cos^2 x) + \sin x - \sin x \cos x = r$$

$$\sin^2 x - \sin x \cos x - 1 = 0 \quad \times r \rightarrow r \sin^2 x - \sin x - r = 0 \quad (*)$$

$$1 - \cos^2 x - \sin x - r = 0 \rightarrow -1 = \sin x + \cos x$$



$$5 \rightarrow r x = r k \alpha + \alpha \rightarrow \left\{ \begin{array}{l} x = \frac{\pi}{r} (rk + 1) \\ x = \frac{\pi}{r} (rk + 4) \end{array} \right.$$

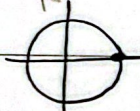
$$\rightarrow r x = r k \alpha + \frac{r \pi}{r} \rightarrow \left\{ \begin{array}{l} x = \frac{\pi}{r} (rk + 4) \\ x = \frac{\pi}{r} (rk + 5) \end{array} \right.$$

$$r - \cos\left(\frac{\pi}{r} + x\right) \cos\left(x - \frac{\pi}{r}\right) = \frac{1}{r}$$

$$\left(\cos \frac{\pi}{r} \cos x - \sin \frac{\pi}{r} \sin x\right) \left(\cos \frac{\pi}{r} \cos x + \sin \frac{\pi}{r} \sin x\right) = \frac{1}{r}$$

$$10 \left(\cos \frac{\pi}{r} \cos x\right)^2 - \left(\sin \frac{\pi}{r} \sin x\right)^2 = \frac{1}{r} \rightarrow \left(\frac{\sqrt{r}}{r} \cos x\right)^2 - \left(\frac{\sqrt{r}}{r} \sin x\right)^2 = \frac{1}{r}$$

$$\frac{1}{r} \cos^2 x - \frac{1}{r} \sin^2 x = \frac{1}{r} \rightarrow \cos^2 x - \sin^2 x = 1 \rightarrow \cos^2 x = 1$$



$$r x = r k \alpha \rightarrow \left\{ \begin{array}{l} x = k \alpha \\ x = k \alpha + \pi \end{array} \right.$$

$$b) \cos\left(r x - \frac{\pi}{r}\right) = -\sin r x$$

$$15 \cos\left(r x - \frac{\pi}{r}\right) = \sin(-r x) \rightarrow \cos\left(r x - \frac{\pi}{r}\right) = \cos\left(\frac{\pi}{r} + r x\right)$$

$$r x - \frac{\pi}{r} = r k \alpha + \frac{\pi}{r} + r x \quad \text{O O E}$$

$$r x - \frac{\pi}{r} = r k \alpha - \frac{\pi}{r} - r x \rightarrow r x = \frac{\pi}{r} (r k - 1 + r) =$$

$$\rightarrow \left\{ \begin{array}{l} x = \frac{\pi}{r} (r k - 1) \\ x = \frac{\pi}{r} (r k - 2) \end{array} \right.$$

$$20 a - \frac{\sin r x + \sin r x}{\sin r x} - \sin^2 x = \cos^2 x$$

$$r \sin r x \cos r x + \sin r x - \sin^2 x = \cos^2 x \quad \sin r x \neq 0 \rightarrow r \cos r x + 1 - \sin^2 x = \cos^2 x$$

$$r \cos r x + \cos^2 x = \cos^2 x \rightarrow \cos r x = 0$$



$$r x = k \alpha + \frac{\pi}{r}$$

$$r x = \frac{\pi}{r} (r k + 1) \rightarrow x = \frac{\pi}{r} (r k + 1)$$

MAHAN

$$9. \frac{\sin \frac{x}{r}}{1 + \cos \frac{x}{r}} = \frac{1}{\sin \frac{x}{r}} + \cot \frac{x}{r} \Rightarrow \frac{\sin \frac{x}{r}}{1 + \cos \frac{x}{r}} = \frac{1 + \cos \frac{x}{r}}{\sin \frac{x}{r}}$$

$$\sin \frac{x}{r} \neq 0 \quad \sin \frac{x}{r} = 1 + r \cos \frac{x}{r} + \cos \frac{x}{r} \rightarrow \cos \frac{x}{r} + r \cos \frac{x}{r} + (1 - \sin \frac{x}{r}) = 0$$

$$\cos \frac{x}{r} \neq -1 \rightarrow r \cos \frac{x}{r} + r \cos \frac{x}{r} = 0 \quad \cos \frac{x}{r} (\cos \frac{x}{r} + 1) = 0$$

$$\text{Unit Circle Diagram} \rightarrow \frac{x}{r} = k\pi + \frac{\pi}{r} \rightarrow x = rk\pi + \pi \Rightarrow x = \pi(rk+1) \quad 5$$

$$-\pi \leq \pi(rk+1) \leq \pi \rightarrow -1 \leq rk+1 \leq \frac{\pi}{r} \rightarrow -r \leq rk \leq \frac{1}{r}$$

$$-1 \leq k \leq \frac{1}{r} \rightarrow k = \boxed{0, -1} \rightarrow x = \pi, -\pi \rightarrow |x_1 - x_2| = 2\pi$$

$$v. \cos^r x - \sin^r x \cos^r x = 1$$

$$(\cos^r x - 1) - \sin^r x \cos^r x = 0 \rightarrow \sin^r x + \sin^r x \cos^r x = 0 \quad 10$$

$$\sin^r x (1 + \cos^r x) = 0 \quad \text{Unit Circle Diagram} \quad x = rk\pi + \pi \rightarrow x = \frac{\pi}{r}(rk+1)$$

$$\textcircled{1} x = k\pi \rightarrow \text{Unit Circle Diagram} \rightarrow \text{Diagram}$$

$$1. \frac{1 - \tan x}{1 + \tan x} = \tan^2 x \rightarrow \frac{\tan \frac{\pi}{2} - \tan x}{1 + \tan \frac{\pi}{2} \tan x} = \tan^2 x$$

$$\rightarrow \tan(\frac{\pi}{2} - x) = \tan^2 x \rightarrow x = k\pi + \frac{\pi}{2} - x \rightarrow x = \frac{\pi}{2}(rk+1) \quad 15$$

$$\rightarrow x = \frac{\pi}{14}(rk+1)$$

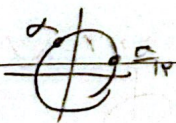
$$9. \sqrt{1 + \sin^r x} = \sqrt{r} \cos^r x \quad [0, \pi] \quad r \cos^r x = 1 + \cos^r x$$

$$\sqrt{(\cos x + \sin x)^r} = \sqrt{r} \cos^r x \rightarrow \cos^r x + \sin^r x + r \sin x \cos x = r \cos^r x$$

$$1 - r \cos^r x + \sin^r x = 0 \rightarrow \cos^r x = \sin^r x \quad 20$$

$$\cos^r x = \cos(\frac{\pi}{r} - x) \quad x = rk\pi + \frac{\pi}{r} - x \rightarrow x = \frac{\pi}{14}(rk+1)$$

$$x = rk\pi - \frac{\pi}{r} + x \rightarrow x = \frac{\pi}{14}(rk+1)$$



$$\frac{\pi}{14} \leq x \leq \frac{\pi}{2}$$

$$x \leq \frac{\pi}{2}$$

$$\text{المعادلة } (\sin^2 n + \cos^2 n)(\sin^2 n - \cos^2 n) = 1 \rightarrow -\cos^2 n = 1$$

$$\cos^2 n = -1$$



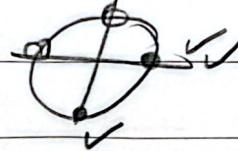
$$r_n = r_1 k + \alpha \rightarrow n = \frac{\alpha}{r_1} (r_1 k + 1)$$

$$\rightarrow n = \frac{\alpha}{r_1}, \frac{r_1 \alpha}{r_1} \text{ بـ } \odot$$

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$$\text{بـ) } (\sin n - \cos n)(\sin^2 n + \cos^2 n - \sin n \cos n) = -1$$

$$\text{بـ) } \sin(n - \frac{\pi}{4}) (1 - \frac{1}{2} \sin 2n) = -1$$



بـ) ١٠٢

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